

DESIGN AND IMPLEMENTATION OF FORWARD AND
INVERSE GRAVITY MODELING SYSTEM

By

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A Project paper submitted to the School of Graduate Studies of Addis
Ababa University in partial fulfillment of the requirements for the Degree
of Master of Science in Computer Science

July, 2008

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SCHOOL OF GRADUATE STUDIES
FACULTY OF INFORMATICS
DEPARTMENT OF COMPUTER SCIENCE

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Abstract

Geophysics is the science that deals with the physical properties of the earth's interior using data obtained from gravity, resistivity, electromagnetic and seismic reflection methods. Among these methods gravity survey is used to detect the underground structure of the earth (density contrast) by means of disturbance they produce at the surface in the earth's gravitational field. Gravity method involves data collection from separated stations in the target area using gravimeter based on relative measurement to a given base station. These collected data are then reduced to remove all quantifiable disturbing effects. From the surveyed and reduced high and low gravity values, it is merely possible to make a tentative and qualitative interpretation, if something is known about the geology.

However, the problem comes from the reduced observations; and from taking into consideration all other available information about the region, how to determine the **size, shape and position** of the subsurface structure giving rise to the gravity disturbance. One of the approaches to analyze and interpret geophysical data is the construction of different geological models. Therefore, in this project we developed an application that helps professionals in the area of geophysics to perform two-dimensional forward/inverse gravity data modeling to acquire some knowledge about the underground geological structure or at least some of its elements, given in terms of the anomalous mass distribution (density contrast) without further destructing the environment. While testing our application on real data that were collected around filweha, the resulting subsurface structural model produced computed gravity data that matched the observed gravity data, within acceptable root mean square error.

Acknowledgement

I sincerely thank my major advisor, Dr. Dida Medikso , for his guidance, and advice throughout this project. Many thanks and appreciations go to Dr. Elias Lewi as all the discussions with him always made me think that things were possible. His eagerness and encouragement has always inspired me to accelerate towards the completion of the work. I would also like to thank all my instructors at the Department of Computer Science, for their personal commitment and contribution to the success of the graduate program.

I am grateful to my friends, Bereket and Gedion, who always shared those harsh and gladly events with me and the same is true for the years to come. In Parallel I would like to express my hearty respect to Etishet who withstand many obstacles for the sake of my success.

Finally, I take this opportunity to express my profound gratefulness to my brothers' one or the other way brought me up to a success in my academic endeavor.

Acronym

SRS Software Requirement Specification

2D Two-Dimensional

3D Three-Dimensional

FA Free Air reduction

GPS Global Positioning System

GUI Graphical User Interface

RMS Root Mean Square

GA Genetic Algorithm

FNN Forced Neural Network

CLS Common Language Specification

cgs Centimeter gram and second

AAU Addis Ababa University

miliGals $\text{mGal} = 10^{-5}\text{m/s}^2$.

1. Introduction

1.1 Background

Geophysics is the science that deals with the physical properties of the earth. Focusing on the solid earth we can say that professionals in the area try to understand the earth's interior using the geophysical data that are collected through gravity, electromagnetic, resistivity, seismic reflection. Among these few technologies gravity method is used to detect the underground structure of the earth to associate variations with differences in the distribution of densities and rock types. The basis of this method is Newton's law of gravity which is stated as every particle of the matter exerts a force of attraction on every other particle, this being proportional to the product of the mass and inversely proportional to the square of the distance between them [12]. The detailed expression is presented in the appendix.

The observed values of gravitational field are totally dependent on the physical properties of the earth at the survey area. Using the highly precise location and elevations pull, gravity method requires measurements of the gravitational field at a series of different locations over an area of interest by taking into account the station spacing to be small enough to prevent a serious aliasing problem. Measurements of gravity fields are of two types, the first one corresponds to determination of absolute magnitude of the gravity at any place. On the other hand, the second approach measures the change in the gravitational field from one place to another place (relative gravity) [11] [14].

A gravimeter together with GPS is used to measure the gravitational field due to any local subsurface structure at any point on the surface based on the second approach. The collected data are then reduced to remove all quantifiable disturbing effects. These deduced data

commonly are known as bouguer gravity anomaly. The Gal (for Galileo) is the cgs unit for acceleration where one Gal equals 1 centimeter per second squared. Because variations in gravity are very small, units for gravity surveys are generally in milligals (mGal) where 1 mGal is one thousandth of 1cm/s².

Before using gravity method as a tool it is necessary to be able to detect spatial change in the vertical gravitational field so as to separate out the change due to the geological structure which is being sought from those of different origin. Only when this is done, that it is possible to consider interpreting the resulting gravity anomaly in terms of geological structure.

Using the surveyed and reduced high and low gravity values, it is merely possible to make a tentative and qualitative interpretation, if some background knowledge about the geology is known. For example, high gravity may be associated with existence of older and denser rocks near the surface or the presence of heavily basic intrusions. Where as low gravity means the existence of sedimentary basin and relatively light acid intrusion [3][4].

However, the problem comes, from the reduced observations, and from taking into consideration all other available information about the region like how to determine the **size, shape and position** of the subsurface structure giving rise to the gravity disturbance.

One of the approaches to analyze and interpret geophysical data consists of constructing different geological models and comparing it with the theoretical geophysical parameters. Among these techniques, we can use gravity modeling which is defined here as a numerical procedure that constructs the structure of the subsurface physical property (density contrast) from measured data and any prior information independent of the measured data. Even if the density of the earth cannot be uniquely determined from gravity, one can often develop a class

of models which will give a 'closest fit 'to the anomaly. Here, we need first to define the mass model in order to compute it using analytical solutions and to carry out quantitative interpretation by drawing geologic conclusions from the created models. In an iterative process one can try to find a best fit of measured and computed gravity curves. For this, either the density parameter of the XY-coordinates or the geometry of the model has to be changed interactively until a reasonable fit between the two is obtained [1][2][5].

Interpretation involving location and delineation of the buried anomaly body-boundary can be performed in two different ways, namely, forward and inverse gravity modeling. In the first approach, the interpreter starts from an initial model M_0 , computes the predicted data values and compares them with the observed one. Then by considering all available information and his intuition, he/she applies correction to the model M_0 in order to minimize the misfits between the calculated and observed data. The procedure is repeated until a satisfactory result is obtained. On the other hand, the second approach comprises all inversion methods to automatically determine the position, the density contrast and the geometry of a causative body, provided that sufficient prior information about source is incorporated by the method [6][10].

1.2 Statement of the problem

Environmental investigations and mineral exploration involve drilling, sampling, and analyzing for anomalous compounds. In other words, they undertake searching and evaluation of concentration of economic metals and other elements found in the natural occurring deposits at or near the surface of the earth. In view of this, geologists have to work through hundreds to thousands of area of cover using every geological, hydrological, geochemical, and geophysical method that are available to assist in the search of resources. In addition to these, successful mining consists of multi- disciplinary activities that require a carefully selected and professional mix from geological, geotechnical, financial, environmental managerial profession [9][13].

Researchers at the Department of Earth Science in Geophysical observatory section, AAU, are being using gravity method as a tool for environmental investigations and mineral exploration. In line with this, analysis of geophysical data requires at least two different geophysical methods. Currently they don't have automated systems, quantitative method for gravity data interpretation. Thus we developed the system that enables them to know about the subsurface structure and maximize the decision support in their study area.

1.3 Objective

The general and specific objectives of the project are described below:

1.3.1 General objective

The general objective is to automate a forward and inversion gravity data modeling.

1.3.2 Specific objectives

The specific objectives are:

- To explore algorithms/methods that can be used for forward and inverse modeling
- To develop a prototype that implements both gravity modeling techniques
- To test the prototype on synthetic and real data.

1.4 Application of the project

According to [11][14] the gravity methods have played increasingly important roles in the search for new reserves of ore (resource exploration like, mineral, geothermal and environmental investigation, etc...) since the development of highly portable gravimeter that has a high degree of precision. Measurements of gravity data provide information about densities of rocks in the earth's underground. For example, a fault is the existence of contrasting rocks of differing densities; hence, the developed gravity modeling application can be applied to find the offsets off the bedrock caused by faulting which might control water of the ascending geothermal system. In general, with the help of gravimeter, the end product of this project will be useful for the researcher to acquire some knowledge about the underground geological structure or at least some of its elements, given in terms of the anomalous mass distribution (density contrast) without further destructing the environment.

1.5 Methodology

In order to develop the system, review of methods and procedures on gravity data modeling techniques has been done. In order to minimize the defect of a system as much as possible, and to validate and verify requirements specification we used interviews, prototyping, use cases class diagram and sequence diagram. The effectiveness of the developed application was tested on synthetic and real data.

1.6 Scope and Limitation of the project

The number of gravity stations, number of blocks in the model, and the number of parameters largely determine the amount of computing time necessary for the program to run. So in this project, interpretation of gravity data was done only for two-dimensional body. On the other hand, the interpretation of gravity data is subjected to limitations. The first limitation is the uncertainty in the source of a given gravity anomaly. The other limitation is when there is no quantitative information of density contrasts between modeled polygons.

1.7 Organization of the Document

This project report contains six chapters including this chapter. Chapter two presents review of research works. In chapter three and four, the paper presents the analysis and design of the developed system respectively. In the remaining chapters, prototype development, and conclusion and recommendations are given.

2. Literature Review

2.1 Overview

Most real geological problems require an answer to a question that goes in the opposite direction, i.e., what combination of material properties and initial conditions may result in this geological response? Hence, the estimation of subsurface properties of the earth by use of observed geophysical data is identified by “**geophysical inversion**”, and can be defined to include nearly everything that geophysicists do. There are a number of subsurface inversion techniques and in each of these cases it is assumed that a physical law holds. In gravitational-field inversion, for example, these laws are described by the gravitational attraction (see the Appendix). With this technique an algorithm based on physical law enables us to invert the observed data for the subsurface characteristics which gave rise to these observations.

Gravity modeling and quantitative interpretation are broadly classified into two major procedures: Data Filtering and Model Fitting

2.2 Data Filtering

Before we proceed to model-fitting, proceeding to model fitting gravity data must systematically be corrected for all factors that influence the magnitude of g at any particular location other than those which represent subsurface densities. The filtered gravity value is obtained after elevation (Free-air) correcting for latitude position on the earth, and the Bouguer correction has been done [6].

2.2.1 *Free-Air Correction*

The free air correction is applied to the station gravity values to remove the effects of the observation points not all being at the same elevation. In short, it will help us to reduce all

station gravity values to a given reference datum station by assuming that there is only air between the observation point and a reference level.

2.2.2 *Latitude Correction*

The absolute gravity value on the earth varies with respect to latitude. The rotation of the earth causes it to bulge at the equator hence it has two effects: (1) an increase in the radius of the earth, and (2) a redistribution of mass.

2.2.3 *Bouguer Correction*

Between any elevated station and the reference datum point there is a thickness of rock exerting a gravitational attraction at the surface which can be considered to be additional to the measured gravity at that station. When reducing an observation down to the reference point, this effect has to be subtracted. Since we are removing from beneath station, a slab of rock of thickness with a specific density as a result will decrease the downward attraction of the free air gravity anomaly. Finally we should compute, Bouguer anomaly, the difference between observed and theoretical gravity at any point on the earth which is purely due to the lateral variation of density beneath the surface.

2.3. Model Fitting

Once we removed the effect of all other factors except the variation in the lateral density contrast of the subsurface from the measured gravity data, the next step is to model the subsurface structure that brought these gravity anomalies. This can be done procedurally in the following manner.

2.3.1 Parameterization

It refers identifying an appropriate list of parameters that can determine the model characteristics such as geometry, density, and depth and station interval.

2.3.2 Selecting Modeling method

It is an approach that is used to decide methods for computing the theoretical data using the identified model parameters. To obtain the gravity effect of a given mass distribution using precise mathematical expression this section discusses some of these methods.

2.3.2.1 The Geometrical Method

The methods belonging to this group approximate the geometrical structure of the bodies of arbitrary shape, first by setting constant density and then the modeling is processed by means of changing the position of the vertices that define the bodies [12]. A similar approach to this method is the one that varies both the geometry and the density of the model selected [13].

Before the actual interactive modeling is constructed, the interpreter must consider how many geological bodies have to be modeled and their position in the vertical planes. These parameters remain constant during the modeling process.

2.3.2.2 Regular shaped Methods

The most popular solution for gravity attraction regular shaped buried body estimation method was given by [5][7]. It considers the earth (the geological volume to be simulated) is composed of many simple elements, for example equally sized rectangular prisms. Because the geometry of the model structure underlying the gravity profile remains constant, only the variable parameters are the densities of the blocks. Hence, the subsurface gravity is then the sum of gravity effect of each cell. By changing density of the selected cells, the interpreter

changes the modeling structure which is assumed to be simple and good in realizing the expected model. Modeling that uses this method can be viewed as model of single parameter (density) which is specified at regular intervals in the subsurface plane with no dependency among these parameters.

The solution of any modeling problem does not depend only on the data quality but also in the choice of model space as well. Hence, when we decide to use any of the methods we should consider how fine these blocks/geometrical shape should be and determine the boundary of the model space, from the total area where the measurements were taken.

2.3.3 Modeling

It is the reconstruction of the model parameters iteratively based on certain criteria so as to obtain a good match with the observed data. There are three main reasons that explain why the final estimated model is different from the real one.

- Non-uniqueness –using gravimeter one can try to estimate the distribution of masses inside the earth, having gravity measure on its surface. It is known from [14] infinitely many different density distributions inside any given body produce identical gravity field on its surface. Therefore, it is not possible to conclude hundred percent about masses inside the earth using only gravity method.
- The real subsurface composition is usually a continuous function of the space but in modeling we limit the number of data to be used. So the choice of data that define the estimated model might have effect on the result.
- The observed data are always contaminated with errors as a result the estimated model is affected by these errors as well.

Modeling-fitting can be performed in two ways, namely, forward and inverse gravity modeling.

2.3.3.1 The Forward Gravity Modeling

The forward gravity modeling procedure consists of a code that computes gravity fields from an assumed subsurface density distribution. However, it requires not only the choice of an appropriate mathematical model, but how many model parameters should be used and which parameters are the most significant. To make it clear, let us denote the forward gravity modeling process as a transformation $f = T(x)$, where f is the model response, x is a vector containing the set of subsurface model parameters, and T is some transformation which we assume will mathematically describe an observed physical process. With this assumption, each geophysical data set is inverted with a forward model selected to simulate the particular physical process producing the recordings. Thus, a gravity simulation algorithm might produce a synthetic gravity field that is to be matched to a set of observed gravity readings. All such algorithms are designed to minimize some measure of the difference between the observed and the computed data [9][10][14].

In order to minimize the difference between the observed and the computed data, most schemes start out with an initial guess of the model parameters and for all successive steps the optimization algorithm yields a set of adjusted or updated parameter estimates. These updated parameters are then "plugged" into the theoretical model, and the resulting new theoretical response should produce an improved match to the data. If this happens, the inversion is said to converge; if not, it is necessary to do these calculations iteratively that is, the above procedure must be applied many times in succession until a satisfactory degree of agreement between the theoretical and the recorded gravity responses has been achieved.

2.3.3.2 *Inverse Gravity Modeling*

Given a set of measured data \mathbf{n} and an algorithm for the problem, inverse modeling will help us to determine the model parameters \mathbf{m} that justify the data using some mathematical or physical basis (typically least squares solution). In other words, it automatically determines the position, density and the geometry of a causative body provided that sufficient prior information about the source is mathematically translated and automatically incorporated by the method.

However, the basic difficulties encountered in an inverse problem are not only the lack of a guaranteed solution or the probable existence of many solutions giving the same answer (non-uniqueness) but also it is "ill-posed". That means, small variations in the solution vector \mathbf{x} can produce large fluctuations in the model response \mathbf{f} , and small fluctuations in the observed data \mathbf{y} can produce large fluctuations in the solution vector \mathbf{x} [6][10]. Thus, the algorithm to be used for gravity data inversion should handle parameterization, measurement and data processing error to avoid problem of instability. Moreover, it should also be flexible enough to accommodate prior information that can avoid the inherent problem of non-uniqueness. This is achieved by performing three conditions: adjusting the solution space, minimization criteria and the mathematical regularization criteria [15].

In general, most of the inversion algorithms that lead to a solution can be grouped either to Bayesian or deterministic approaches. In the Bayesian approach, one assumes that a prior density function can be assigned for the model and tries to solve the problem by maximizing the posterior probability of the model [3][4][7]. On the other hand, the deterministic approach

assumes that there is no prior information and tries to solve the problem by using as much information as possible from the measured data.

Both approaches try to find solution for the problem that has several local minima through an iterative application to achieve “global optimization”. The following sections discuss how those algorithms work.

2.3.3.2.1 Compact gravity inversion algorithm

In the work of [15] the density of rectangular prism was considered as a weighting matrix (W_m) to get the compact subsurface mass distribution. The inversion procedure converges to compact model that is not necessarily single density; and mostly it will give large density model that is not realistic. To overcome this problem, the author suggested to put a density constraint that can limit the upper bound of the density. Then any block that crosses the density barrier (X_o) will be set to X_o and the algorithm automatically freezes this block in the next iteration by assigning a very small weight to this block.

2.3.3.2.2. Genetic algorithm (GA)

The author in [10] developed this search algorithm which is suitable for the inversion of highly non-linear functions. Starting with a set of random solutions, this algorithm progressively modifies the solution set by imitating the evolutionary behavior of biological systems (selection, cross-over, and mutation), until an acceptable result is achieved. Unlike other classes that optimize one single solution, GA belongs to a class of algorithms that work by optimizing collection of solutions. During the inversion process, the GA applies parameter swapping between highly-ranked models to generate a new set of models that progressively converge towards the target geological section. As in biological evolution, an element of

randomness exists in the generation of new models, so that unexpected results may suggest new possibilities outside the experience or expectation of the geologist.

2.3.3.2.3 Forced Neural Network

According to [8] neural network modeling system is the system that simulates the ability of neurons to self organize and learn in respect to the external parameter. The artificial neural network is composed of many simple processing elements, which are massively interconnected and operate in parallel. The processing elements commonly known as neurons receive input from previous elements and send the output to other elements through synaptic connections. These connections have different weights. In order to find the effective values of inputs and outputs, these values are multiplied by these weights. The main purpose of neural networks is to compute such weights giving the best output. To obtain the eligible values for weights, they use back propagation learning algorithm for neural networks, The objective of the learning process is to adjust the free parameters of the network to minimize error. To do this minimization, they used the Least Mean Square (LMS) algorithm and considered a simple method of training in which the weights are updated on a pattern-by-pattern basis until one epoch, that is, one complete presentation of the entire training set has been dealt with.

While generalizing inverse modeling algorithms, they will leave little or no room for interactive supervision by the user if there is a chance to quantify everything with mathematical expression. But, this computing method, without a human arbiter, is a dream.

In summary, the forward gravity modeling needs iterative numerical trial and correct error procedures, which in turn consume time and difficult for complicated problem. Whereas, in

inverse modeling it is often difficult to quantify all qualitative knowledge of experts, and we will end up with conceptual geological targets which cannot be adequately described by numerical data. This is in contrast to, for example, the inversion of gravity profile, where the gravity data are the objective measure we are trying to reproduce. Thus, this calls for a possible combination of the methods for a better result.

So the principal objective of this project is to combine the trail and correction modeling with an inversion modeling in order to interpret gravity anomaly. This is achieved by developing a gravity modeling application that uses the rectangular block approach to compute gravity anomaly of a given density structure.

3. System Analysis

Taking into account the requirement gathering techniques this chapter presents the functional and the non-functional requirement of the system:

3.1 Functional requirements

It defines the specific functions, tasks and behavior of the system. In light with this, the developed system is expected to provide the following functionalities:

- i. The system should be able to generate synthetic data
- ii. The system should have a way to record field data
- iii. The system should be able to perform data filtering
- iv. The system should be able to generate model space and parameters
- v. The system should be able to do forward and inverse gravity modeling

3.2 Non-Functional Requirements

- **Performance (Response Times)**-The the system should react and display what is expected on relatively few time
- **Usability (Human Factor)** - The system must be able to do most of the tasks so that clients can make minimum human interaction.
- **Reliability (Frequency of Failure)** -The system should be available and function at all times with a minimum downtime.

3.3 Model Analysis

For the purpose of this project we have described the analysis model in terms of use case view, behavioral view, and structural view as follows.

3.2.1 Use Case Model

The above functional requirements can be expressed within the “Use-Case Model”, for better understanding of requirements elicitation. The main concepts of use case modeling are *actors* and use cases. An actor represents an entity (human or non-human) external to the system under development that communicates with the system in order to achieve certain goals. Here interpreter (geophysicist) belongs to the primary actors. They can interact with the system by starting and stopping the system (human-machine interaction) and this interpreter has user goals to be provided by the system. On the other hand, use case describes a sequence of actions that provides something of measurable value to an actor. Figure 3.1 depicts the use case diagram of the system.

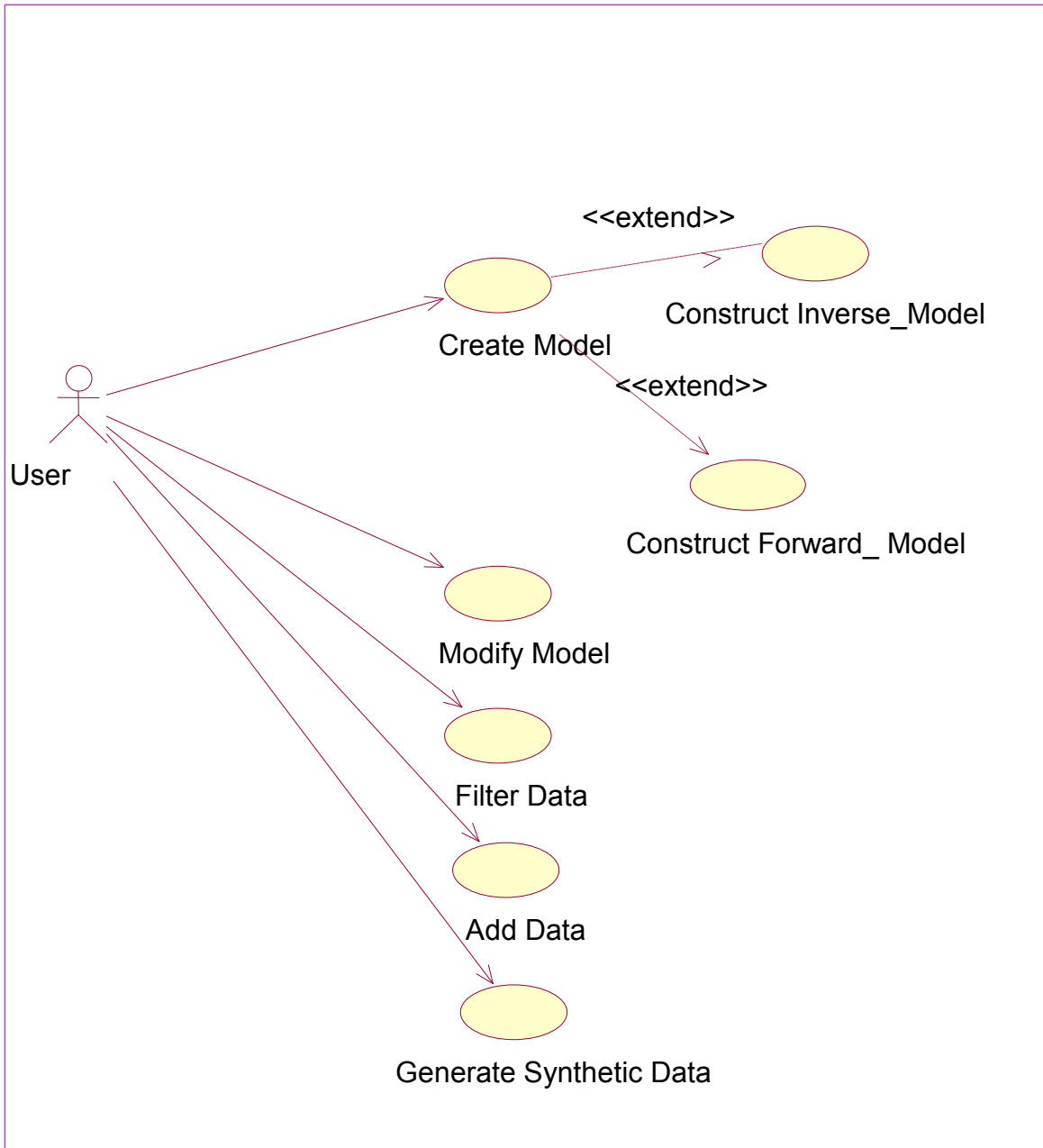


Figure 3.1 : Use case Diagram

The following list of tables describes the primary actor and sequences of steps needed to be performed so as to accomplish the identified use cases in Figure 3.1.

Table 3.1: Description of construct forward model use case

1. Use Case Name ConstructForwardModel

Actor : Interpreter

Pre condition : There must be a file that contains filtered gravity data

Flow of Events :

- i. The use case starts when a user clicks on CreateNewModel button
- ii. The system requests the user to select the filtered data file. [Alternative A1]
- iii. The system requests the user to input another input parameters like , prism width, height, depth, density range of the survey area.[Alternative A2]
- iv. The system verifies the validity of input values , if they are valid the system generates the main interface window that contains
 - List of interactive menus
 - Subsurface model space
 - Model response display area[Alternative A3]
- v. The user selects prism from the model space and changes its parameter value. [Alternative A4]
- vi. The system executes the modified model response based on the modified values

vii. The system saves the current model parameters

Post condition
Alternatives

The system saves the final model response and ends the use case

- A1: The system displays error message if it is unable to get an appropriate file format and returns to the beginning state
- A2: The system displays error message and returns to the beginning state.
- A3: The system alarms the user to input valid input values and redisplay the input Form
- A4: The system displays parameter value out of range error message and redisplay the input Form

Table 3.2: Description for construct inverse model use case

2. Use Case Name :	ConstructInverseModel
Actor :	Interpreter
Pre condition :	There must be a file that contains filtered gravity data
Flow of Events :	<ol style="list-style-type: none">i. The use case starts when a user click on CreateNewModel buttonii. The system requests the user to select the filtered data file. [Alternative A1]iii. The system requests the user to input another input parameters like, prism width, height, depth, density range of the survey area. .[Alternative A2]iv. The system verifies the validity of input values , if they are valid the system iteratively compute model parameters and displays the corresponding model response
Post condition	The system saves the final model response and ends the use case
Alternative	
A1:	The system displays error message if it couldn't get an appropriate file format and reset itself to the beginning state.
A2:	The system alarms the user to input valid input values and redisplay the input Form

Table 3.3: Description of Data Filtering use case

3. Use Case Name :	FilterData
Actor :	Interpreter
Pre condition :	There must be a file that contains station information
Flow of Events :	<ol style="list-style-type: none">i. The use case starts when a user clicks on DataReduction buttonii. The system requests the user to select the station file. [Alternative A1]iii. The system executes data reduction and displays the result.iv. The system saves the data.
Post condition	The system acknowledges the user and ends the use case
Alternative	
A1:	The system displays error message if it is unable to get an appropriate file format and returns to the beginning state

Table 3.4: Description of use case that generate artificial data

4. Use Case Name : GenerateSyntheticData

Actor : Interpreter

Pre condition : The system must be started

Flow of Events :

- i. The use case starts when a user clicks on GenerateSyntheticData button
- ii. The system requests the user to input data generating criteria.
[Alternative A1]
- iii. The system generations synthetic data
- iv. The system saves the data

Post condition The system acknowledges the user and ends the use case

Alternative

A1: The system alarms the user to input valid input values and redisplay the input Form

Table 3.5: Description of use case that can help to modify existing model

5. Use Case Name :	ModifyModel
Actor :	Interpreter
Pre condition :	There must be a file that contains previously done model response
Flow of Events :	<ol style="list-style-type: none">i. The use case starts when a user clicks on OpenOldModel buttonii. The system requests the user to select the file that contains model parameters.[Alternative A1]iii. The system accepts further modification of model parameters and displays the corresponding output.iv. The system saves the data
Post condition	The system acknowledges the user and ends the use case
Alternative	
A1:	The system displays error message if it encounters problem while reading the existing model information and reset itself to the initial

Table 3.6: Description of use case that perform data recording

6. Use Case Name :	AddData
Actor :	Interpreter
Pre condition :	The system must be started
Flow of Events :	<ol style="list-style-type: none">i. The use case starts when a user clicks on AddData buttonii. The system displays data recording formiii. When the user finishes data entry he/she will press save data
Post condition	The system saves the data and ends the use case.[Alternative]
Alternative	The system displays error message if it encounters data mismatch while saving to the persistent data storage

3.3.2 Behavioral View (Sequence Diagram)

A sequence diagram is used to show and capture the interaction between participating objects in a given use case [16]. It is also helpful to identify the missing objects that are not identified in the analysis object model. The following diagrams describe the direct translation of identified use cases in to the sequence diagram.

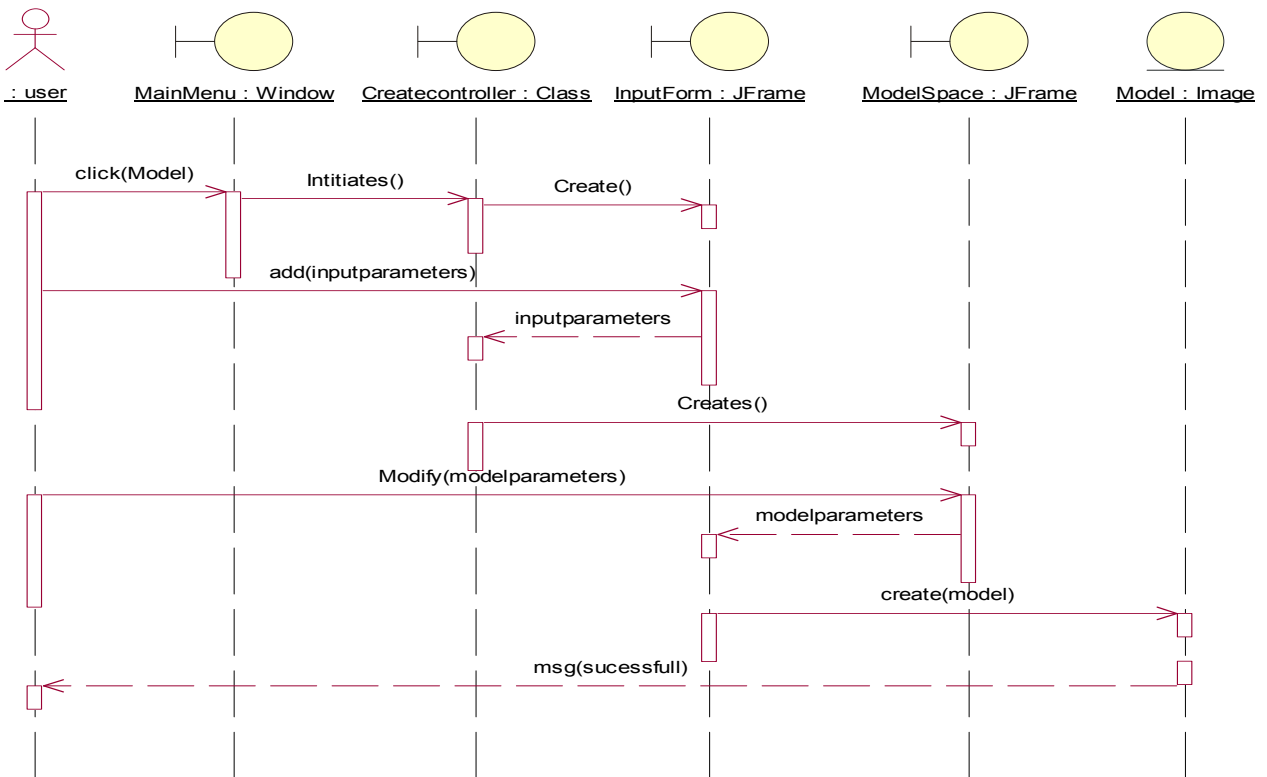


Figure 3.2.: The sequence diagram for the forward gravity model generator

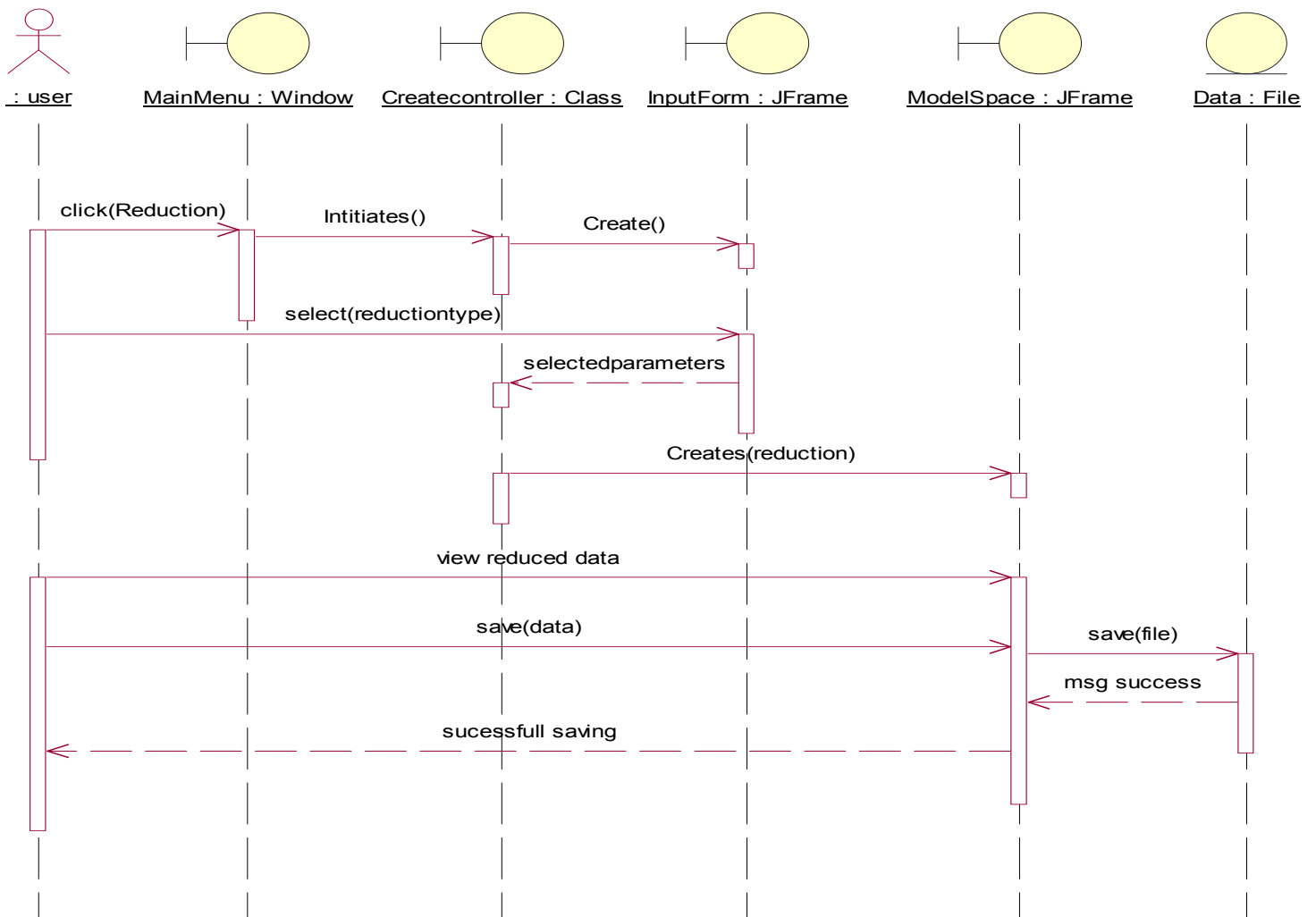


Figure 3. 3: Sequence diagram for data filtering

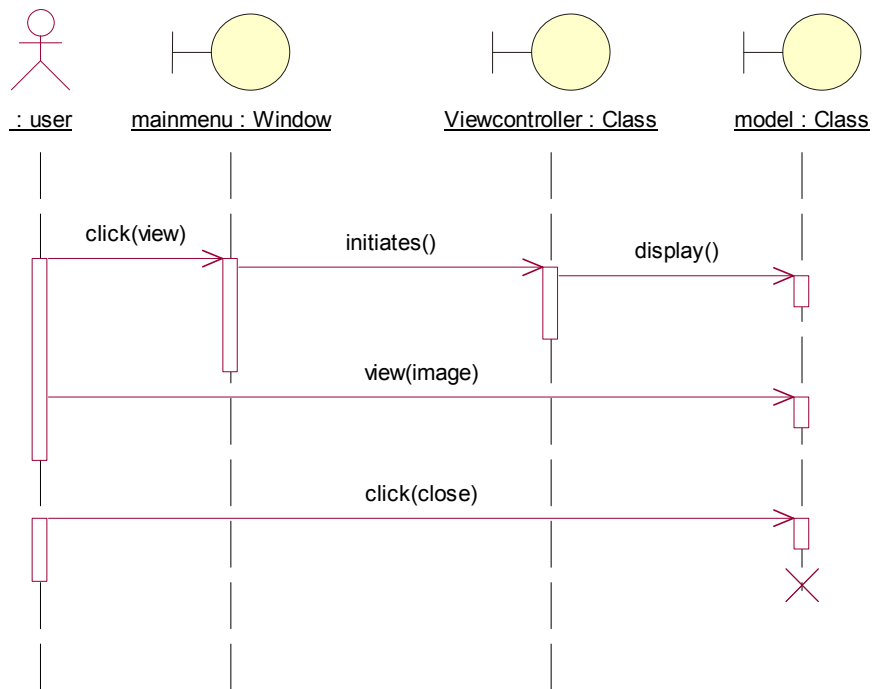


Figure 3.4: Sequence diagram for viewing an existing model

3.3.3. Class Diagram

The following class diagram depicts the set of classes that are identified in our system.

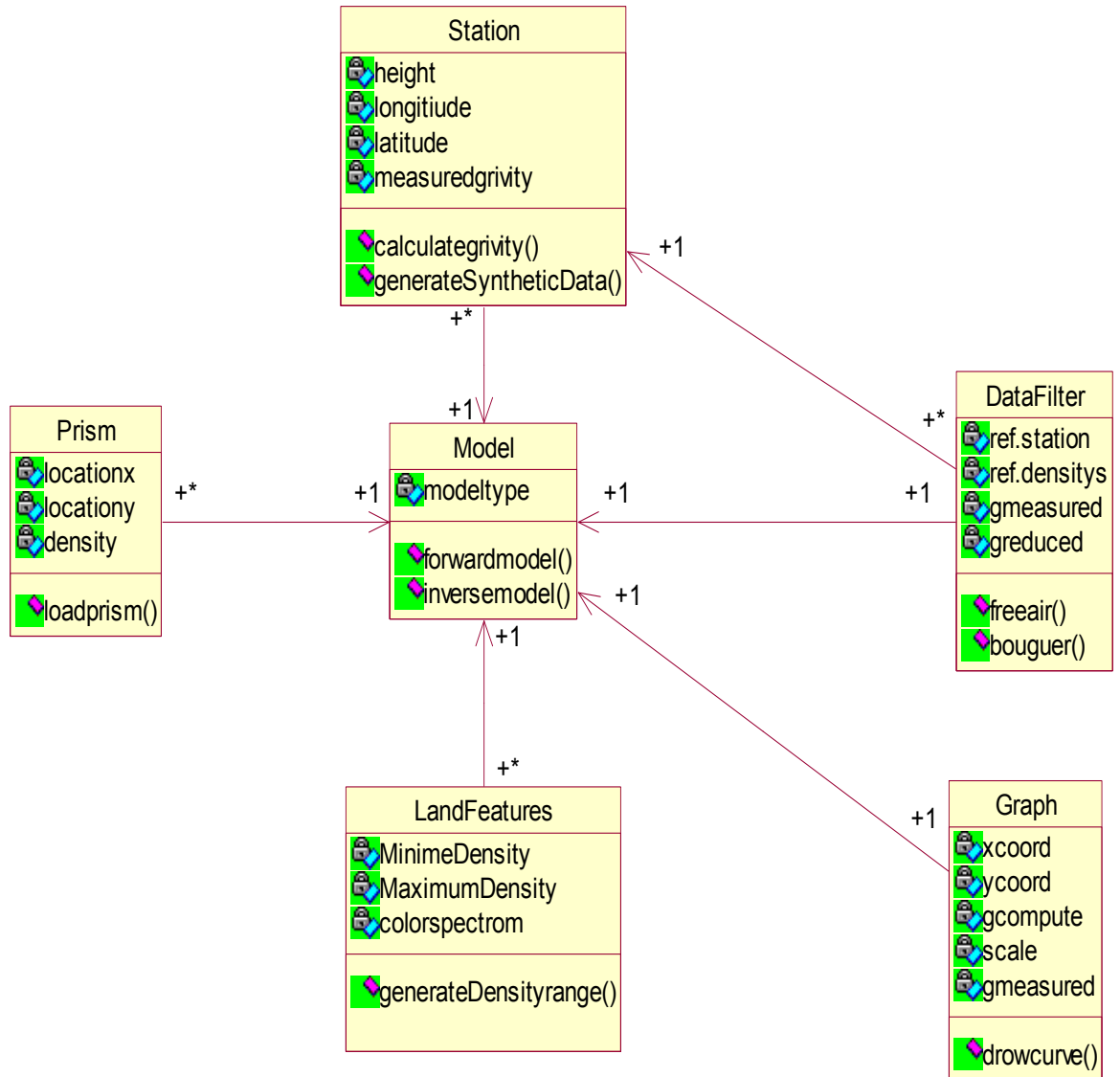


Figure 3.5: Class Diagram

Description of classes:

- **Station** – this class is responsible for describing about spatial distribution of stations. Each station is defined by nine values: it's Station Name, Station No, distance from the reference point, latitude, longitude, Height, Observed and calculated gravity values.
- **Prism** – it is responsible for providing subsurface information in terms of blocks used in modeling: each block is defined by seven parameters: four spatial coordinate density values of a block and RGB Color spectrum.
- **LandFeatures** - provides information about maximum and minimum density contrast values of the study area , density scale, Reference density value and RGB Color Spectrum
- **DataFilter**- it is responsible for performing data filtering based on reference density, reference station on the measured gravity data
- **Graph**- it is responsible for displaying the effect of a given model structure
- **Model**- is responsible to estimate model parameters through forward and inverse modeling techniques so as to bring the similarity between measured and theoretical gravity data.

4. System Design

4.1 An overview of system design

The series of activities that will be performed during gravity data interpretation may look as follows: by means of modeling software, the interpreter creates an initial density model using his knowledge about the region of investigation. The initial model is then sent to the inversion program, the model resulting from the inversion is then analyzed and the eventual corrections are applied and repeated until a satisfactory result is obtained.

Interpretation of gravity data by model fitting requires the interpreter to alter the parameters of an assumed model until a good visual fit is obtained. In this process the parameters of an assumed model are varied systematically by an algorithm until the model 'data' matches the observed data in some least squares sense. To end up with this, the data structure which is required for the description of the two dimensional model, geometry must be simple and flexible enough to cover the wide field of gravity modeling: It should facilitate representations of geological information, such as vertical or horizontal cross sections, depth, as well as volume and mass calculations. These requirements lead to the structures (geological bodies) to be modeled are bounded by list of rectangular prism, which limit domains with constant density. Using this data input methods for our system also minimizes parameterization problem and it is good enough to estimate the shape of irregular buried bodies. For our modeling techniques it was assumed that all blocks are of the same dimension and the subsurface geometry will not be altered for each iteration.

By taking into account the above design objective we discuss below some of the basic issues that we have seen how we performed the system design.

The analysis model which was mentioned in the previous chapter describes the system completely from the actors' point of view which does not contain information about the internal structure of the system, its hardware configuration, or how the system should be realized. However, system design is the transformation of the analysis model into a system design model with the aim of decomposing the system into smaller subsystem that can be realized at the time of implementation [20]. During system design and implementation phase of the system, we try to fill the gap between system specification which is produced during requirement elicitation and analysis, and the final software product. To do this, we first set the design goals and then the architecture of the system in terms of different components of the system as discussed as follows.

4.2 Design goals

The definition of design goals is the first step of system design and it identifies the qualities that system should focus on. We derived them from the non- functional requirements and set the following three major criteria that our system should satisfy.

i. Maintenance criteria

The difficulty of modifying and deploying the developed system to the target machine was addressed in the following manner:

Modifiability- The system is built of several more or less independent classes which can be used as a standalone application or replaced by other classes. This makes the system easy to change the existing functionality or add new ones when the need arises.

Portability -Once the system is developed, the deployment to the target machine will be carried out by creating its executable file (Published one) which makes system to be easily portable to different platforms.

ii. **Dependability criteria**

- **Robustness**

Ability to survive invalid user input is assured during data input and generation of model parameters by providing some information about the error and then the system resets itself to the previous safe state.

- **Reliability**- in order to maintain the difference between specified and observed system behavior we try to test it as much as possible

iii.**End user criteria** -include qualities that are desirable from users' point of view that have not yet been covered under the performance and dependability criteria.

- **Utility** -The system must address the possible functional requirement of the users
- **Usability**-The System should be user friendly, and easy to learn and use

4.3 Software architecture

We identified to use repository software architecture since we have only single data structure which is residing on a single machine. We also addressed some of the Software architecture components like, subsystem decomposition, subsystem mapping to hardware, data storage, and inter subsystem communication.

4.3.1 Subsystem Decomposition

In Chapter 3, Modeling with UML, in order to reduce the complexity of the system analysis (Application domain) we identified smaller parts called classes. Similarly, to reduce the complexity of the system design (solution domain), we decompose a system into simpler parts, called subsystems, which are made of a number of solution domain classes. A subsystem can be represented as a directory containing all the files implementing the subsystem with a set of related operations that share common purpose so as to provide service to other subsystem. While performing subsystem decomposition we considered the two basic concepts namely coupling and coherence. Figure 4.1 shows the subsystem decomposition.

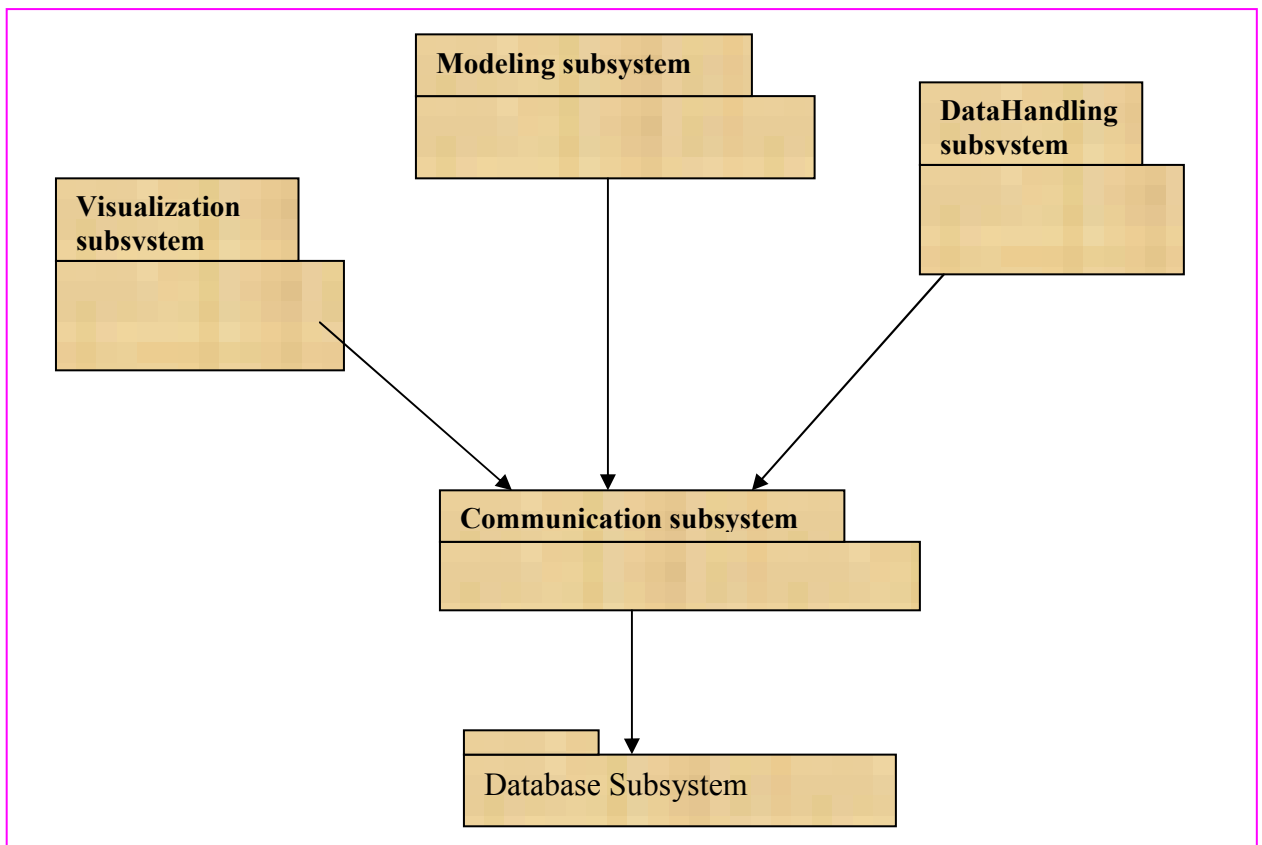


Figure 4.1: Subsystem decomposition and their dependencies

Description of each subsystem is given as follow:

- i. **DataHandling Subsystem** - is responsible for handing all the necessary model parameters and data, data recording, data viewing, data generating, through its DataInterface. It contains two main classes namely Station and Prism.
- ii. **Modeling Subsystem** – it is responsible for making data filtering, performing forward and inverse modeling

Based on the concept which was described in chapter two, the effect of changes in elevation, latitude and land topology on the gravitational field is done using the equation given in the appendix.

We also designed the system for the forward and inverse gravity modeling based on the following concept and algorithm

- **Forward Model**

In order for forward gravity modeling to compute the vertical gravity anomaly at a point due to an arbitrary body shaped two-dimensional (rectangular prism) we used the algorithm which is developed by [7]. i.e let the horizontal and the vertical distance from the center of the reference point to the center of the j^{th} rectangular prism be X_j and Y_j respectively. If the width and the height of the rectangle are W and H, respectively, then the vertical gravity anomaly at a measuring point i due to the j^{th} prism can be given as:

$$g_i = a_{ij} \rho_j \text{-----} (4.1)$$

Where

- ρ_j - the density of the j^{th} rectangular block
- a_{ij} - is the geometric effect of the j^{th} rectangular block at the i^{th} station.

The detail about forward gravity modeling expression is presented in the appendix.

• **Inverse modeling**

In many of gravity exploration, a single density of model is desired, as the exploration objective is often a body of a specific mineral ore, whose density is essentially constant.

Thus we therefore attempt to find models in which the density contrast of individual cells is either zero or a constant value in the hope of defining a simple anomaly body.

To solve this problem we used a formula developed by [15] which assumes the subsurface mass distribution is divided into smaller regular shaped bodies in which the density within each block is constant with desirable geological characteristics such as compactness and uniform density contrast. Using matrix notation it is rewritten as follows:

$$\bar{X} = W_m^{-1} A^T \left(A W_m^{-1} A^T + \frac{\delta_m^2}{1 + \delta_e^2} \right)^{-1} G \text{-----} (4.2)$$

Where

G- The data vector whose elements are the data that are filtered at a point

A- Matrix whose j^{th} element contains the vertical gravitational attraction of the j^{th} block with a unit density on the i^{th} measuring point

A^T - The transpose of matrix A

\bar{X} - A vector that contains M number of unknown parameters

W_m^{-1} - The parameter weighting matrix

δ_m^2 - The variance of the parameter values

δ_e^2 - Variance of the deviation between filter and computed gravity anomalies

More information about the 2D data inversion is presented in the appendix.

- iii. **Visualization subsystem**- it is responsible to draw the theoretical and filtered gravity value based on the result obtained from the modeling subsystem.
- iv. **Communication subsystem**- it is responsible to handle interaction among subsystems through global variable.
- v. **Database subsystem**- it is responsible for storing and retrieving data from the data base. All of the subsystems identified here have direct contact with this sub system.

4.3.2 Hardware/Software mapping

Next to subsystem decomposition we considered where to put these subsystems. Issues for such type of components of software architecture can be addressed by asking question like; what is the hardware configuration of the system? Which node is responsible for which functionality? Computers are modeled as nodes in UML deployment diagrams which are used to depict the relationship among run-time components and hardware nodes. Thus we selected a single window machine as the virtual machine on which the system will run. Once the hardware configuration has been defined and the virtual machines selected, subsystems are assigned to nodes. Figure 4.2 shows the deployment diagram of the subsystem.

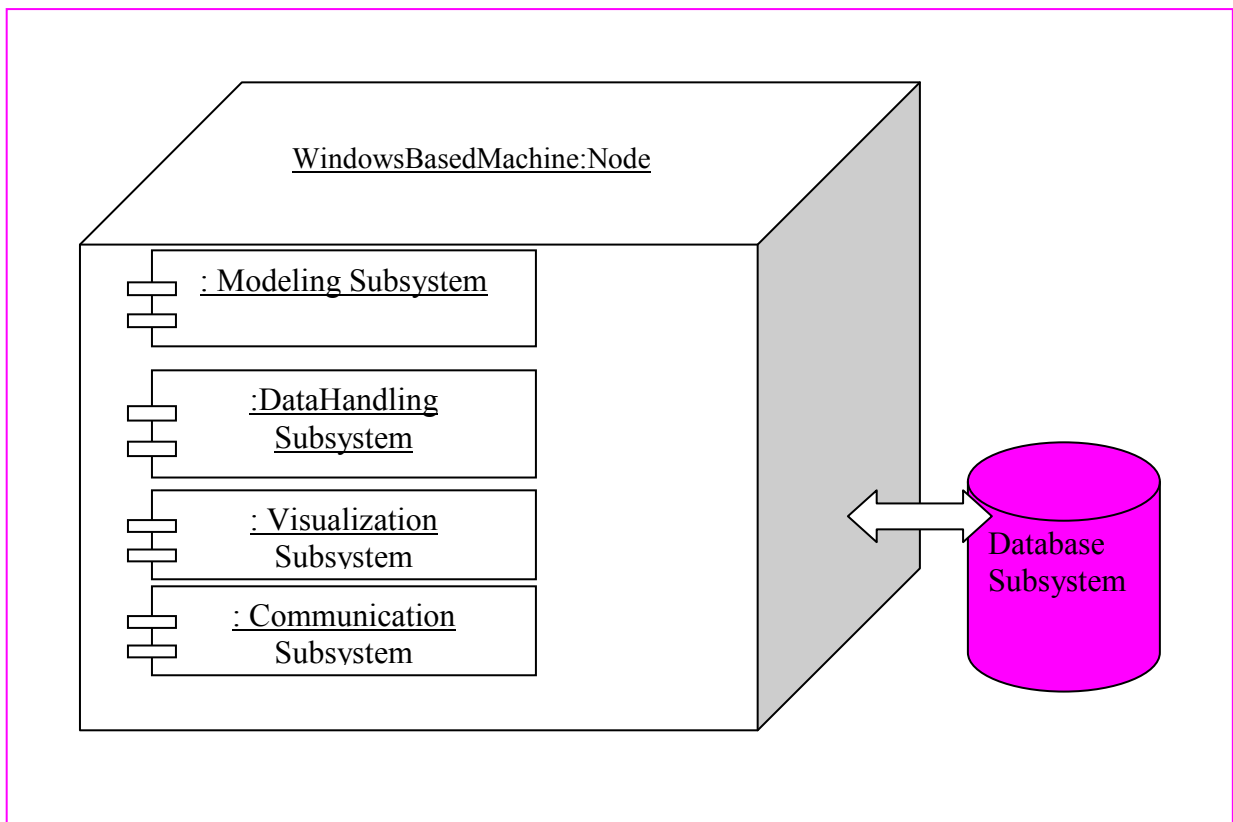


Figure 4.2: Deployment diagram that show allocation of subsystem at the running node

4.3.3 Persistent data management

Most functionality in our system is concerned with creating or manipulating data. For this reason, access to the data should be fast and reliable. If retrieving data is slow, the whole system will be slow. So in this system component description we address issues like, which data need to be persistent? Where should persistent data be stored? How are they accessed? To answer these, we identified the persistency of objects directly from the application domain and then we decided how these objects should be stored. By considering the benefits of Flat File data storage system [16] i.e. , its data management abstraction is relatively low level and enable the application to perform a Varsity of size and speed optimization; appropriateness of its storage system when we are dealing with low information density, we selected it for temporary and persistent data storage and management.

5. Implementation

5.1 Programming tools

.NET Framework used to develop applications and services like, Console applications, Windows GUI applications (Windows Forms), ASP.NET applications, XML Web services. In addition to these common tasks, the class library includes types that support .NET Framework collection classes which implement a set of interfaces that we can use to develop our own collection classes. So by taking into consideration the services provided by .Net framework, we developed the system with this environment using C# programming language.

5.2 Prototype

The main document interface (MDI) window is the home window which will be displayed when the application is started. This window includes all subsystem interfaces which are organized in a menu format. Figure 5.1 depicts this window.



Figure 5.1: Main Window

Once the main window is activated, the users of the system can get data recording functionality through the data recording form. Figure 5.2 shows the data entering form. The same form is also used to view an already existing and filtered data.

SName	SNo	Position	Longitude	Latitude	Height	Measured Gravity	Normal Gravity	Calculated Gravity	Free Air Correction	Bouguer Correction
0082	992017	20	49	9.23	2045	976782.12543				

Figure 5.2 : Data Recording Form

Figure 5.3 shows a form which is responsible to handle user’s preference to make reduction to a given station.

Figure 5.3: Reference Station Input Form

When a user initiates to perform data reduction by clicking menu option under data reduction category, the system executes request and displays the result using the Form given in Figure 5.4.

SName	SNo	Position	Longitude	Latitude	Height	Measured Gravity	Normal Gravity	Calculated Gravity	Free Air Correction	Bouguer Correction
0082	990082	1	38.740709	9.018617	2064	977468.68188	978159.55927	0	38.65301	1.17359
0023	990023	69	38.740751	9.018122	2066	977468.30558	978159.54545	0	38.90773	1.2044
0001	990001	179.3	38.740897	9.017187	2068	977467.70608	978159.51936	0	38.95152	1.02429
0002	990002	260	38.761449	9.016606	2067	977466.82819	978159.50315	0	37.78124	-0.03404
0007	990007	308.5	38.761768	9.016271	2066	977467.23924	978159.4938	0	37.89304	0.18971
0010	990010	370.8	38.762117	9.015642	2060	977467.11538	978159.47625	0	35.93513	-1.09648
0094	990094	491.8	38.762452	9.014753	2052	977468.62417	978159.45145	0	34.99992	-1.13606
0017	990017	607.2	38.762513	9.013906	2046	977470.90812	978159.41946	0	35.46426	0

Figure 5.4: Free air and Bouguer gravity anomaly

Figure 5.5 represents a form which is used to accept user's preference in splitting the subsurface earth structure in terms of prism width, prism height and depth. The form is also responsible to handle user's interest in unit of measurement. The density contrast range controls are used to capture information about the distribution of rocks density in the study area.

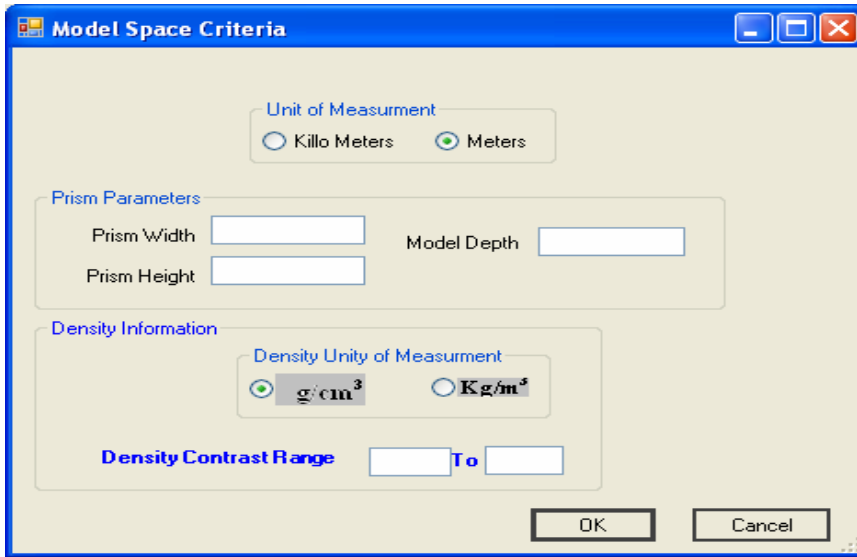


Figure 5.5: Subsurface Parameter Input Form

To set up the forward model, a station file must be first available to the system and then the subsurface information is generated automatically. The structures generated by the program have the following feature: all blocks at the same depth have same height; and width of a block can not vary with depth or along the profile. The proper size of the block has a direct effect on the quality of the interpreting relatively small bodies. For example, if the size of the block is too big, the resulting structure will be rough. On the other hand, the use of two small blocks will generate large amount of prisms and makes the modeling process a tedious task .The model resolution and the number of observation points influence the physical size of the model. Hence, one should keep in mind that the size of the model and the time needed to calculate its gravity anomaly is proportional to the product of the number of stations and prisms.

Once the said conditions are settled the system generates a new window which looks like in Figure 5.6

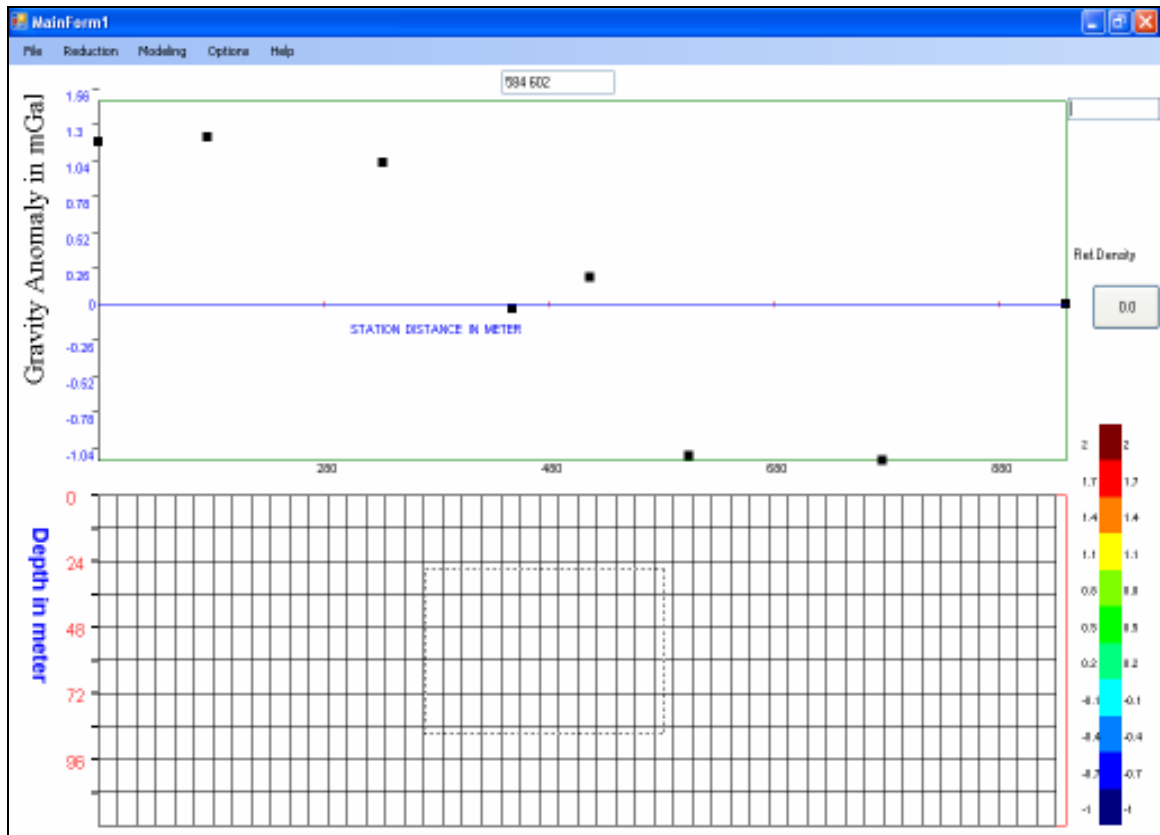


Figure 5.6: The Initial Forward Modeling Window

The initial forward modeling window has two active sub windows: the partitioned subsurface representation, the bottom one, and the graph area, the upper one. The small black dots stand for the actual filtered gravity values at each survey station that is supposed to provide information about the distribution of subsurface structure during inversion. The small rectangles that are arranged in a row and column manner indicate the subsurface division. These rectangles were generated from a reference point in the study area along the earth surface and below this reference point to a certain depth. The list of numbers and color spectrum, density contrast, color, and density of substance, that are arranged to the right side of this window are legend which can indicate the possible types of substances in the study area and what types of substances are buried under a given modeling process.

In order to start the forward gravity data modeling, the interpreter first selects certain prism(s) and changes the density of these prisms and observes the effect. If there is a significant gap between the forward modeling result and the actual gravity data, the interpreter continues to do modification. After a number of modification the fitted model may look like as shown in Figure 5.7

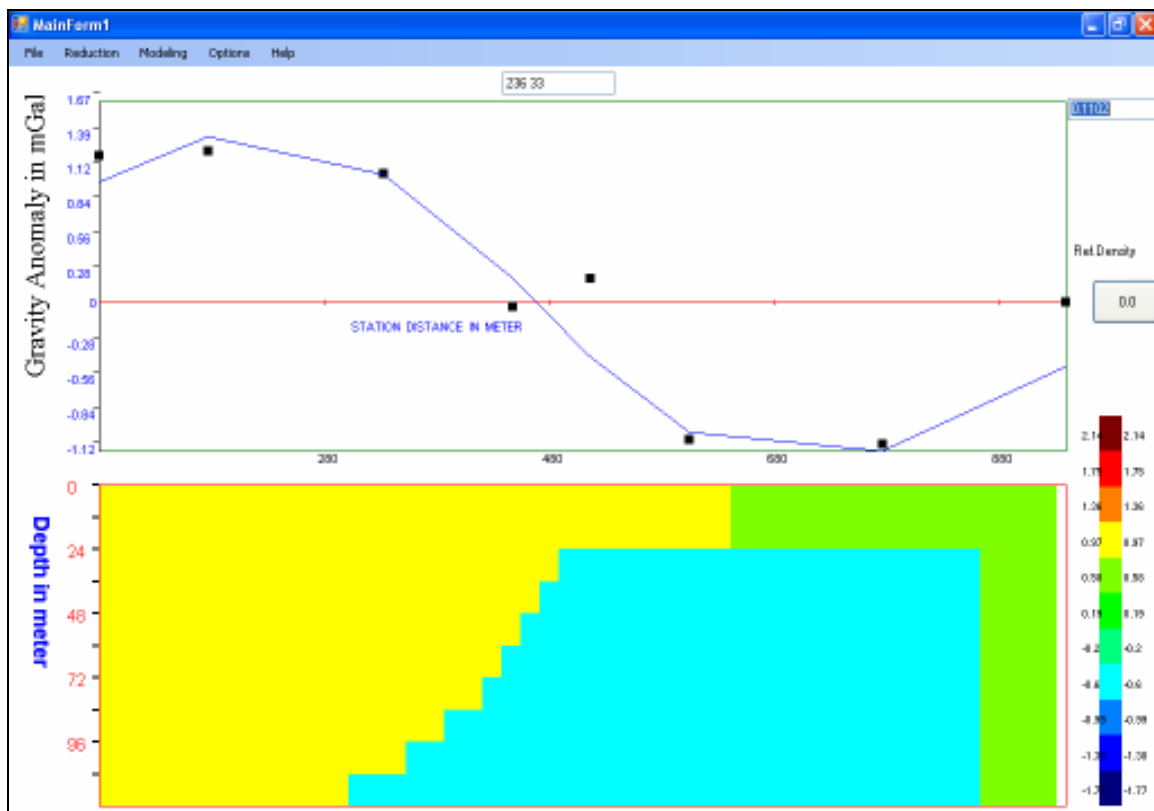


Figure 5.7: Modified version of Figure 5.6

The inverse gravity modeling program was also written to better determine the subsurface structures in the modeled area. However, this iterative solution technique requires an initial guess of parameters. "Bad" initial parameter guess may cause a parameter totally outside of the area of assumed location. If this occurs, successive iterations will result in the divergence of parameters from the true solution. Therefore, a careful choice of the initial parameters is necessary to achieve reliable results.

In order to check the result of an inverse model, we first generate artificial data by assuming that there is some single body inside the subsurface like in the figure 5.8. The curve in this figure indicates the gravity effect of this buried body. We then gave these gravity effects of the buried body to the inverse model. Figure 5.9 depicts the resulting estimated buried body-boundary along with the misfits at a particular iteration.

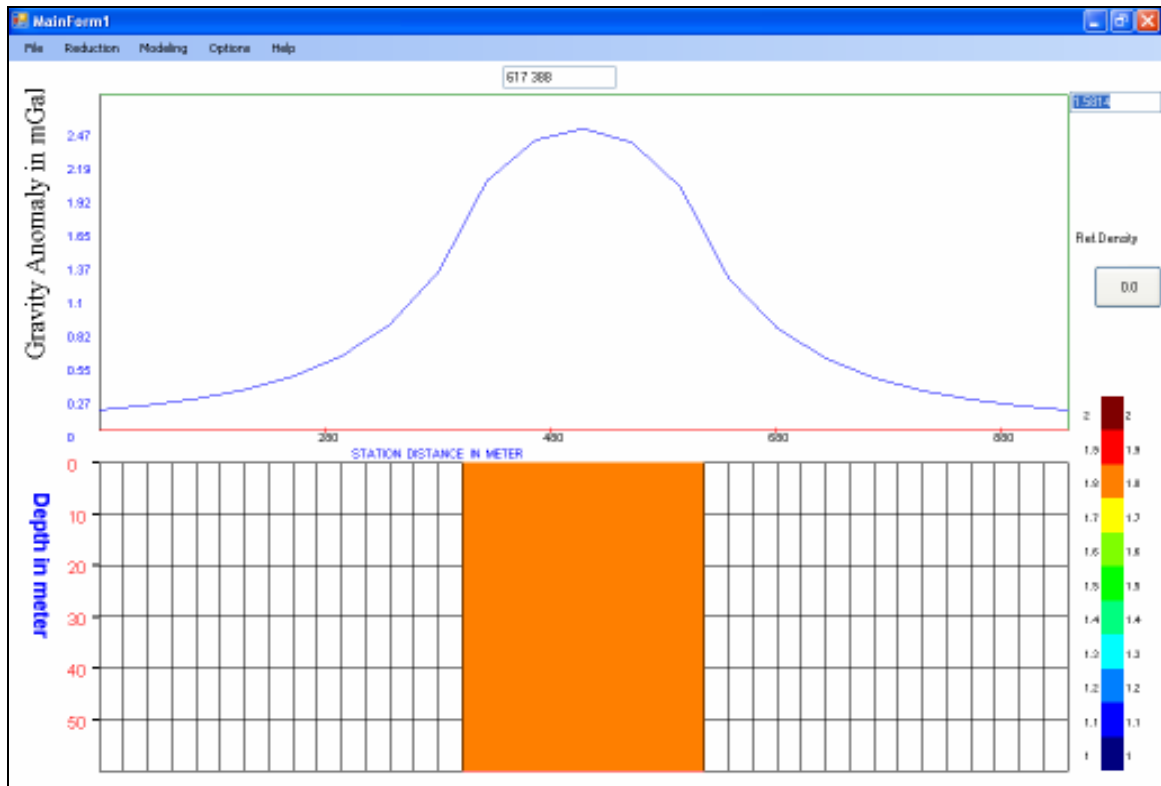


Figure 5.8: Synthetic Data (assuming that the colored body is buried)

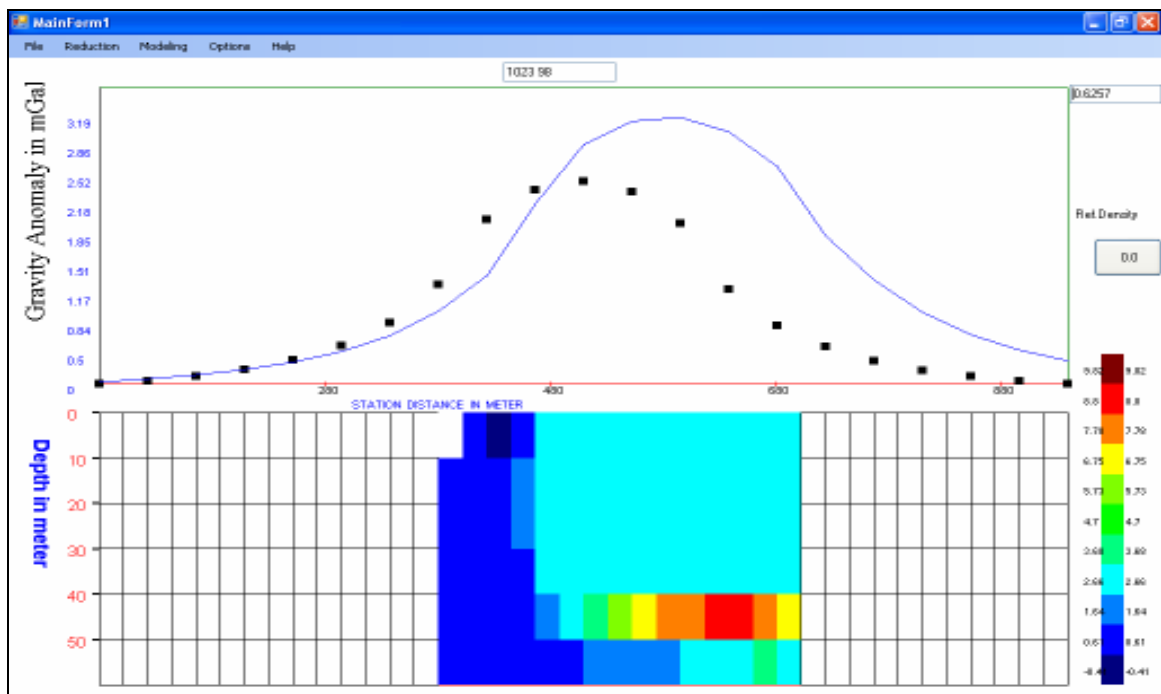


Figure 5.9: Estimated buried body boundary for Figure 5.8 using inverse model.

5.3 Tests

We tested our application software in order to see its effectiveness on different synthetic and real data. Synthetic data were generated randomly and the two inversion techniques were applied on these data. As a result, the response obtained by the combination of the two techniques was found to be near the randomly fabricated data within the predefined root mean square error. We also used real gravity data around filweha area (around Sheraton Addis) that have different geologic settings and borehole (drilled subsurface data). The purpose of testing our system in the area of filweha hot spring was to determine the location of faults which have offset the bedrock and which might possibly control the ascending thermal waters of the filweha geothermal system. In this case our system also showed acceptable subsurface structure with that of the expected geological settings.

6. Conclusion and Recommendation

6.1 Conclusion

The geophysical interpretation of gravity data is a complicated and often an intensive task that requires a lot of experience from an interpreter. On the other hand, interactive modification of model parameters and direct visualization of both computed and measured fields of gravity data enable the interpreter to design the subsurface structure as realistic as possible. A basic requirement for modeling is the existence of ideas and hypothesis on the investigated area, i.e. the availability of quantitative or qualitative constraints.

Towards this end, we discussed the steps in processing and interpretation of gravity data: Parameterization, and a forward and inverse gravity data inversion in terms of the causative bodies. To minimize parameterization problem, we assumed the representation of the subsurface of the earth into a set of small rectangular blocks. However, limitations of computer power and memory forced us to implement the model for 2-D bodies which in turn suppressed the application of rectangular blocks methods in modeling of high resolution density structures.

The forward modeling is the most important technique in gravity interpretation in some simple cases. However, in order to speed up the interpretation significantly and to overcome the limitation of this modeling technique, we combined the forward and inversion methods. Nevertheless, an inversion needs appropriate target parameter which totally depends on the interpreter's prior knowledge about the study area. The simplest and most effective constraining method is a well prepared (by means of the forward modeling) starting model. The model resulting from the inversion is only one of the possible solutions; but instant

visualization of the resulting structures enables fast estimation of the model quality and application of eventual corrections. All operations were done with Dot Net framework and through successive application of the method the program was tested on several 2-D synthetic and real gravity profiles data.

6.2 Recommendation

We used gravity survey that was collected horizontally labeled single profile data. We think that the model can be enhanced further if one considers the following:

- Developing a system that handles gravity data that are collected in a grid form so as to consider the effect of 3-Dimensional buried bodies.
- Developing a subsystem that can draw the contour map (IsoGal) of gravity data which is good in providing qualitative information.

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Appendix

1. Newtons law of attraction

$$F = G M_1 M_2 / R^2 \dots\dots\dots (1)$$

where

F – The force of attraction between two particles;

G- The universal gravitational constant, $6.67259 \times 10^{-11} \text{ M}^3 \text{ Kg}^{-1} \text{ S}^{-2}$

M- The unit of measurement for distance in Meter

Kg- the unit of measurement for mass of particle in kilogram

S- The unit of measurement for time in second

M₁ , M₂ - Represent the mass of two particles;

R- The distance separating the masses.

By the 2nd law of motion, gravitational acceleration at the earth's surface by a body of mass **M₁** due to the attraction of mass **M₂** is derived by dividing both sides of equation (1) by **M₂**, and letting **M₁** equals the mass of the earth, we can get the equation for gravitational acceleration:

$$F/M_2 = g = G M / R^2 \dots\dots\dots (1)$$

2. Gravity data reduction type

- *Free-Air Correction*

We used the technique in [8] to find the free air gravity gradient by assuming the earth is sphere as

$$g_{FreeAir} = 0.3086h$$

Where h is station elevation.

- **Latitude correction**

Latitude correction using the method in [11] at the position on the reference ellipsoid as:

$$g_n = g_e (1 + \alpha + \beta + \gamma)$$

Where

g_n - represents gravitational acceleration at any point on the surface of the earth

g_e the gravity on the equator above sea level= 978032.53359

α = 0.0052790414 * Sin(Latitude²)

β = 0.0000232718 * Sin(Latitude⁴)

γ = 0.0000001262 * Sin(Latitude⁶)

- **Bouguer Correction [14]**

$$g_h = 0.4093\rho h$$

Where ρ represents the density of material in g/cm³, and h is the thickness of slab in meters.

$$g_{Bouger} = g_o - g_n + g_{FreeAir} - g_h$$

Where g_o is Gravity readings observed at each gravity station h

3. The mathematical formula used for the forward modeling

$$a_{ij} = 2 * G * \left[\left(X_i + \frac{W}{2} \right) \log \left(\frac{r_2 r_3}{r_1 r_4} \right) + W \log \left(\frac{r_4}{r_3} \right) - \left(Y_j + \frac{H}{2} \right) (\theta_4 - \theta_2) + \left(Y_j - \frac{H}{2} \right) (\theta_3 - \theta_1) \right]$$

Where

W - Prism width

H - Prism Height

X_i - The horizontal distance from the center of the reference point to the center of the rectangular prism

Y_j - The vertical distance from the center of the reference point to the center of the rectangular prism

$$r_1^2 = \left(Y_j - \frac{H}{2} \right)^2 + \left(X_i - X_j + \frac{W}{2} \right)^2, \quad r_2^2 = \left(Y_j + \frac{H}{2} \right)^2 + \left(X_i - X_j + \frac{W}{2} \right)^2$$

$$r_3^2 = \left(Y_j - \frac{H}{2} \right)^2 + \left(X_i - X_j - \frac{W}{2} \right)^2, \quad r_4^2 = \left(Y_j + \frac{H}{2} \right)^2 + \left(X_i - X_j - \frac{W}{2} \right)^2$$

$$\theta_1 = \arctan \left(\frac{X_i - X_j + \frac{W}{2}}{Y_j - \frac{H}{2}} \right),$$

$$\theta_2 = \arctan \left(\frac{X_i - X_j + \frac{W}{2}}{Y_j + \frac{H}{2}} \right)$$

$$\theta_3 = \arctan \left(\frac{X_i - X_j - \frac{W}{2}}{Y_j - \frac{H}{2}} \right),$$

$$\theta_4 = \arctan \left(\frac{X_i - X_j - \frac{W}{2}}{Y_j + \frac{H}{2}} \right)$$

Declaration

I, the undersigned, declare that this project is my original work and has not been presented for degree in any other university, and that all sources of materials used for the project have been acknowledged.

Declared by:

Name: _____

Signature: _____

Date: _____

Confirmed by advisor:

Name: _____

Signature: _____

Date: _____

Place and date of submission: Addis Ababa, July 2008.