Addis Ababa University

School of Graduate Studies

Department of Computational Sciences

Air Traffic schedule: The case of Bole International Airport.

A Thesis Submitted to the School of Graduate Studies in partial fulfillment of the requirements for the Master of Science degree in Computational Science.

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Air Traffic schedule: The case of Bole international Airport

By

Thomas Wetere

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Chairman of department or Graduate committee   Signature

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Examiner's Signature   Signature

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Declaration

This is to certify that the thesis entitled **" Air Traffic schedule: The case of Bole international Airport ",** submitted by Thomas Wetere to the Graduate school of Addis Ababa university for the award of the degree of Master of Science in computational science is a record of research work carried out by me under the supervision of **Tilahun Teklu(PhD).** The contents of this thesis, in full or in parts, have not been submitted to any other Institute or University for the award of any degree or diploma.

By: Thomas Wetere

Signature: _______________

Date: ____________________

Adviser: Tilahun Teklu (PhD)

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Date: __________________

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List of Abbreviations

OR -Operation research

λ -Arrival rate

μ -Service rate

Wq -Average waiting time in the queue

Lq -Average length of the queue

L-Average length in the system

CDF-Cumulative distribution function

GD -General discipline

SIRO: Service in Random Order

FCFS: First come first served

LCFS: Last Come, First Served.

SIRO: Service in Random Order.

PD: Priority Discipline.

Abstract

This thesis is limited to the presentation of the single-server waiting line systems with Poisson arrivals and exponential service times. Waiting systems are stochastic mathematical models and they represent the describing base of the waiting phenomena, service processes and appropriate performance measures. The objective here is to describe the state of waiting line phenomena at Bole international airport and then improve its performance measurements. The data collected from Ethiopian civil aviation authority is then analyzed using mathematical models of queuing theory to determine performance characteristics of the Airport under steady state. Finally, solution results show that performance characteristics of Bole international Airport can be improved by either increasing the service rate or by adding new servers (runways) so that more aircrafts can be served simultaneously. Based on these results, recommendations are provided.

Key words: Waiting line Models, queuing theory, waiting system, waiting line, Utilization rate, arrival rate, service rate, and queue discipline.

1. Introduction

Air Traffic Control has become a safety issue of great importance during the last decade because of the many near-miss or tragic accidents that have occurred worldwide. The main reasons for these accidents are:

***** Air Traffic Control system failure,

***** Air Traffic Control erroneous procedures,

***** Pilot error,

***** Weather conditions,

***** Increased air traffic etc

This issue has even greater significance at each circumstance because of the large volume of aircraft activity in the air in specific areas highly attributed to waiting line phenomena.

Waiting lines and service systems are important parts of the business world and the most common phenomenon in our daily life. The study of waiting lines, called queuing theory, is one of the oldest and most widely used quantitative analysis techniques.

Waiting lines are an everyday occurrence, affecting people shopping for groceries buying gasoline, making a bank deposit, or waiting on the telephone for the first available airline reservationists to answer. Queues, another term for waiting lines, may also take the form of machines waiting to be repaired, trucks in line to be unloaded, or airplanes lined up on a runway waiting for permission to take off or to land ,the physical queue for service such as might be found in one’s local post office, bank, supermarket or cinema etc. The three basic components of a queuing process are arrivals, service facilities, and the actual waiting line.

The most important issue in waiting line problem is to decide the best level of service that the organization should provide. For example, to cope up with the aircraft reservation queue, how many counters must be opened? If the counters are too less, there will be very long queue, resulting in long waiting time. This results in dissatisfaction among the customers. However, if the service counters are too many the counters may remain unoccupied for quite some time. This would result in loss to the service organization. An important issue to understand in queuing

problem is about arrival pattern. Generally, the arrival of customers is random, which may be depicted by a probability distribution. Besides this, the arrival may also be influenced by hours of a day, season, days of month, etc. For example, the arrival of people to watch film at cinema is more at afternoon hours as compared to that of morning. The management may open more or less service counters, depending upon the arrival; but extra counter means additional cost in running this service. As there are Computers which are very fast, accurate, programmable, good in storage are now very common, a reduced and dependable service time is what customers and management want. Fairness is, indeed, the key characteristic of the queuing protocol

The crucial issue in waiting line is to provide a compromise between good service (by less service time) and less cost in running the service points. In overall, the total expected cost of service, which is the sum of providing service and cost of waiting time, is expected to be minimum at certain service level. This service level should be optimal service level.

1. 1 Conceptual background

1.1.1. Queue in the Operations Research (OR) Perspective

When the demand for a particular product or service exceeds the available capacity, a waiting line is formed. Reasons for this may be a shortage of available products in stock or servers, or there may be limitations to the available space where the service or product is provided. To understand the true problems behind, one must know how much service or products should be made available that is reflected by factors such as the average length of waiting, number of people in a waiting line, service rate, …etc. All these factors can be taken into account with the use of the queuing theory, which will be discussed in the following section.

1.1.2. Queuing Theory

Elements of Waiting Line Analysis

Waiting lines form because people or things arrive at the servicing function faster than they can be served every single time. This does not mean that the service function is understaffed; rather it means that waiting lines form because customers do not arrive at a constant rate, nor are they all

served in an equal amount of time. A waiting line is continually increasing and decreasing in length, and in the long run approaches an average rate of customer arrivals and an average time to serve the customers.

Decisions about waiting lines and the management of waiting lines are based on the averages for customer arrivals and service times. They are used in queuing formulas to compute **operating characteristics,** such as the average number of customers waiting in line and the average time a customer must wait in line.

Different sets of formulas are used to calculate operating characteristics, depending on the type of waiting line system being investigated.

Elements of a Waiting Line

The basic elements of a waiting line, or **queue**, are

- arrivals
- servers
- the waiting line structure

Following is a brief description of each waiting line component.

The Calling Population

The calling population is the source of customers to the queuing system, and it can be either infinite or finite.

- Infinite - a large enough population that one more customer can always arrive to be served
- Finite - a countable number of potential customers.

The Arrival Rate

The **arrival rate** is the rate at which customers arrive at the service facility during a specified period of time.

  - It expresses the frequency of customer arrivals at a waiting line system.
  - It typically follows a Poisson distribution.
  - Average arrival rate = λ

Service Times

Service is expressed in terms of *time*, but it can be converted to a rate to be compatible with the arrival rate. Service can be represented by a number of probability distributions, but it typically follows the negative exponential distribution.

- Service times often follow a negative exponential distribution.
- Average service rate = μ

Queue Discipline and Length

Queue discipline is the order in which customers are served. First come, first served is the most common, but random and last-in, first out is possible in some manufacturing or service systems. Queues lengths can be infinite or finite.

  - Infinite is most common.
  - Finite is limited by some physical structure, like a driveway that can accommodate only a limited number of cars.

Basic Waiting Line Structures

Waiting lines are generally categorized into four basic structures, according to the nature of the service facilities: single-channel, single-phase; single-channel, multiple-phase; multiple-channel, single-phase; and multiple-channel, multiple-phase. (See figure below)

- o **Channels** are the number of parallel servers.
- o *Phases* denote number of sequential servers the customer must go through.

Operating Characteristics

Operating characteristics are the criteria that can be used to evaluate the performance of a queuing system. They are descriptive, not optimal decision results.

- Mathematics of queuing theory does not provide the optimal or best solutions.
- Computed operating characteristics describe system performance.
- Steady state is the constant, average value for performance characteristics that the system will reach after a long time.
- The results of operating characteristic calculations should be used to evaluate if the performance of the system satisfactorily satisfies customers and company policy.
- Typical operating characteristics computed are:

Notation Description

L- Average number of customers in the system (waiting to be served)

L_q -Average number of customers in the waiting line

W- Average time a customer spends in the system (waiting and being served)

W_{q-} Average time a customer spends waiting in line

P_{0-} Probability of no (zero) customers in the system

P_{n-} Probability of *n* customers in the system

$\rho-$ Utilization rate; the proportion of time the system is in use

(1.1.3.4) Queuing Disciplines

(i.) Order of Service

This specifies the order in which customers were chosen for service within a queue.

Among the disciplines under this category

☐ ***FCFS:*** First Come, First Served. This is the most commonly used discipline applied in the real world situations, such as check-in counters at the airport.

☐ ***LCFS:*** Last Come, First Served**.** This illustrates a reverse order service given to customer versus their arrival.

☐ ***SIRO:*** Service in Random Order.

☐ ***PD:*** Priority Discipline. Under this discipline, customers will be classified into categories of different priorities.

(ii.) Structure of the Queue

This specifies the physical setup of the queue, which combines two main factors: the number of servers and lines available. Among the disciplines under this category

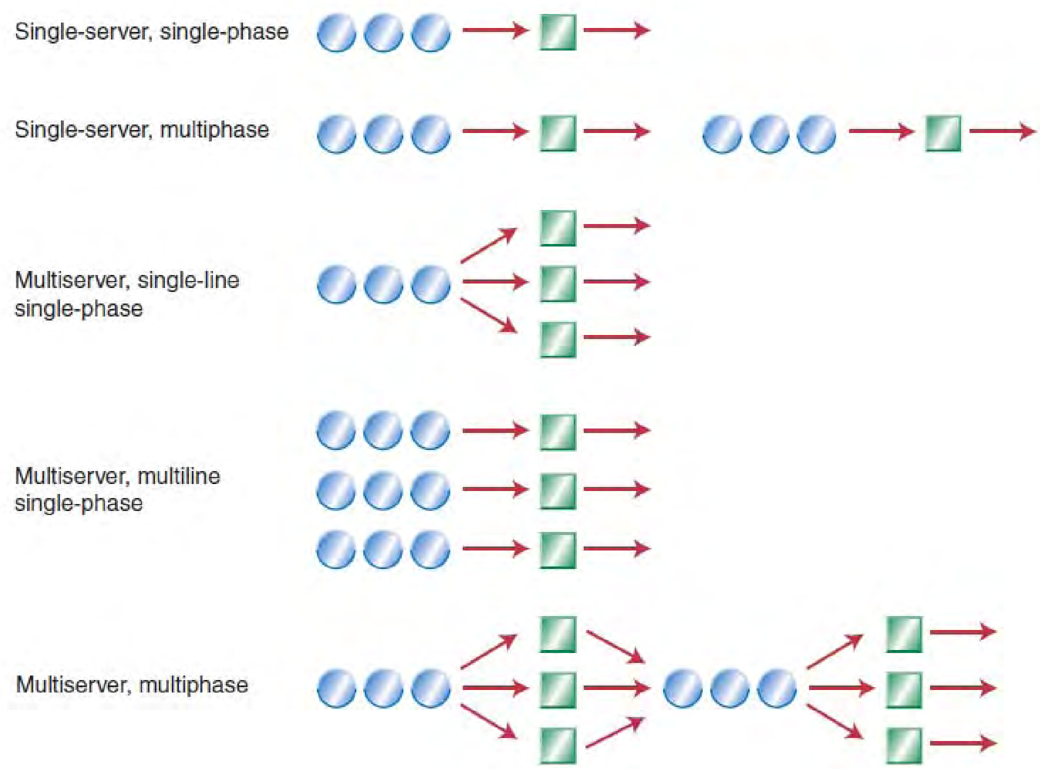

Fig1.Basic waiting line structure

1.1.4 Distribution of arrivals

A feature of the arrival process is the probability distribution of arrivals in a given time period. In many situations, arrivals occur *randomly* and *independently* of other arrivals, such that the estimation of an arrival occurrence is difficult to determine. Thus, the Poisson distribution is the best model to describe the arrivals pattern.Starting from the definition of a Poisson distribution of random variables [, the probability distribution function of *x* arrivals in a specific time period, is:P{x=k} = $(\lambda^k e^{-\lambda})/k!$

where: x – the number of arrivals in a time period;

λ – the mean number of arrivals per time period; e = 2.71828…

Example

The probabilities of 0,1, & 2 aircrafts arrivals during a 1 hour period a t Bole airport are:

$$P(0)= \frac{(6)^0 e^{-6}}{0!} = 0.0025$$

$$P(1)= \frac{(6)^1 (e^{-6})}{1!} = 0.0150$$

$$P(2)= \frac{(6)^2 (e^{-6})}{2!} = 0.0446$$

- Poisson probability distribution provides a good description of the arrival pattern. In a tabular form:

No of arrivals	probability (Poisson)
0	0.0025
1	0.0150
2	0.0446
3	0.0892
4	0.1339
5 or more	0.1606

1.1.5 Distribution of Service time

Service time starts when a customer places an order and finishes when the customer receives the order. Service time is not constant, but it depends on how large the order is.

Exponential distribution of the service time provides the best information regarding the operations of waiting line. If the exponential probabilistic distribution is used then the probability that the service time is less than or equal to a time *t* will be given by

$P(t_s \leq t) = 1 - e^{-\mu t}$ where: *ts* – the time for service;

t – the length of the specified time period;

μ – the mean number of items that can be served in a period; $e = 2.71828\ldots$

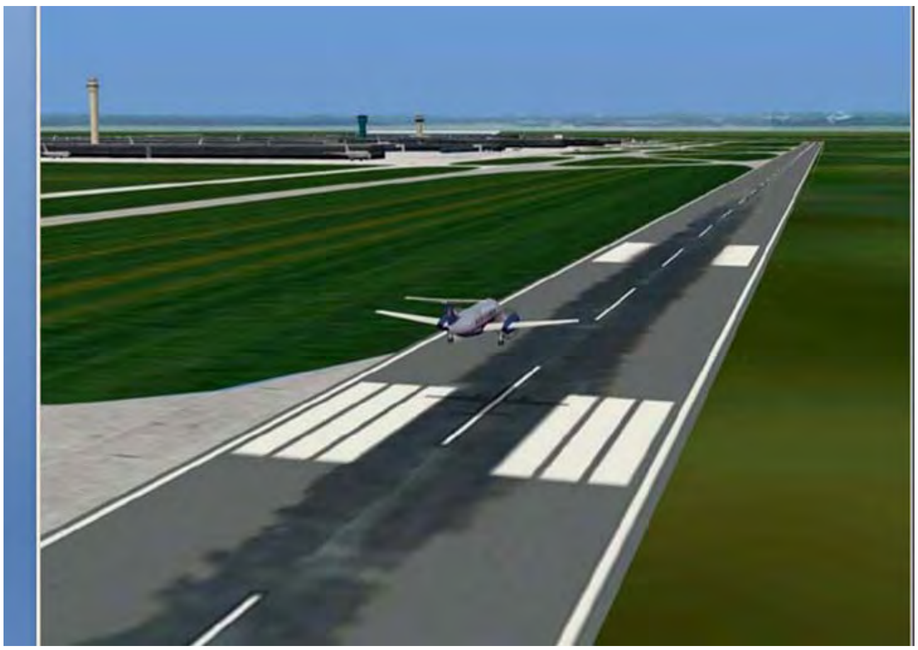

Figure 2. Runway

1.2. Statement of the problem

Waiting in lines is part of everyday life .One of the most important managerial applications of random processes is the prediction of congestion in a system, as measured by delays caused by waiting in line for a service. Customers arriving at a bank, a checkout counter in a clothing store, a theater ticket office, Aircrafts to land, Aircrafts to take off, the Registrar's Office for class registration, a supermarket checkout, etc. may perceive that they are wasting their time when they have to wait in line for service. Repeated and excessive delays may ultimately influence the customers' service preferences. If the wait is too long, the customer may be dissatisfied or balk. If one thinks about the lines he /she have waited in just every day it has a fundamental role in bringing the customers-years of useful work each day. Waiting is a result of the number of customers served, the number of servers working, and the amount of time it takes to serve each individual customer. Waiting line phenomena can also be observed on Aircrafts at airports. Of the airports available in Ethiopia Bole international airport can be cited as a preluding. In Bole international Airport the case of waiting by aircrafts is widely seen as there is only one runway. This in turn brings a number of problems on the organization in providing optimal service. Due to this conducting a study is rational to minimize the problem to a certain degree. Shortly and precisely, to secure the most economical strategy in improving services and to bring optimal service level.

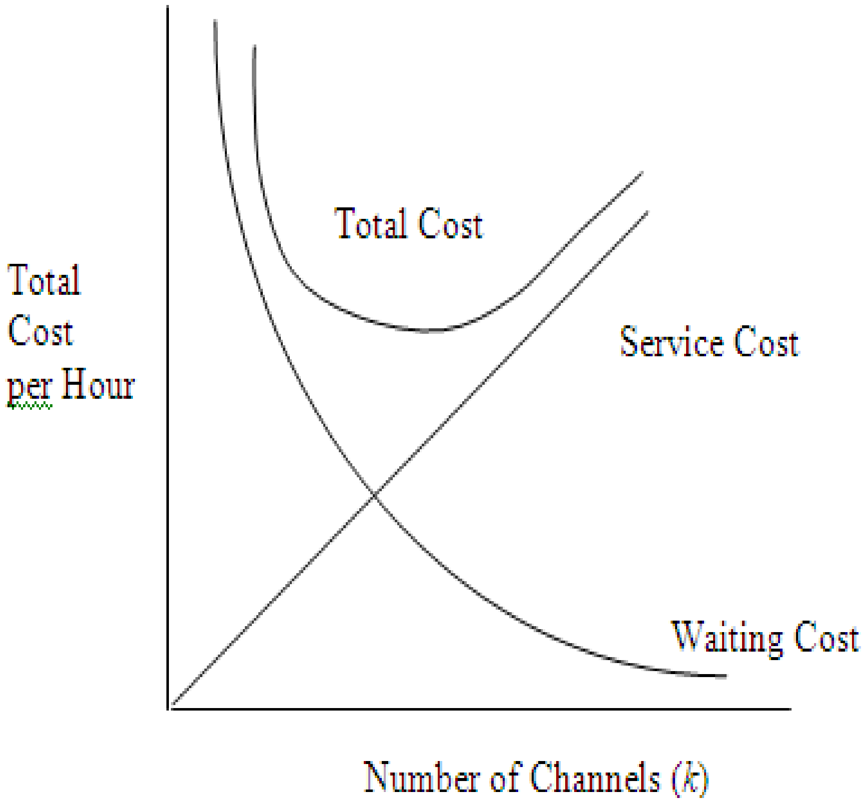

Fig.3. The general shape of waiting cost, service cost and total cost curves in waiting line model.

1.3. Purpose of the study

1. 3.1. General Objective:

The objective of this research is to describe the state of waiting line phenomena at Bole international airport and then to improve the cost-effectiveness and flexibility of air traffic operators and to prove the effectiveness of those techniques in a case study and finally promote the safety, efficacy and consistency of international air transport in Ethiopia.

1.3.2. Specific Objective:

a. Minimizing the waiting time of aircrafts.

b. Develop computer modeling

c. Increase system and technique to utilize resources

d. Satisfying the need of the customer related to time .i.e. reducing alleviation of traveler frustration.

e. Managing traffic flow

f. Evaluating performance measurements of Bole international Airport.

1.4. Limitations of the study

The following are the limitations of this study.

• The unavailability of data because security reasons and the like

• All aircrafts are assumed to have poisons arrival and exponential service time.

• The researcher was also limited to resources of related literatures.

• The researcher assumed that there is no balk.

• The researcher assumed that there is no reneging

• The researcher assumed that there is no jockeying.

• When the real life situations are portrayed by an analytical queuing theory model various assumptions creep in.

• The queue discipline is first come first served.

• The conditions done are all under steady state.

• Queue models are complex to handle analytically when the number of server increases.

• The researcher got shortage of budget allocated for the study.

1.5 Definitions and terms

Single-channel queuing system-A service system with one line and one server

Multiple-channel queuing system-A service system with one waiting line but with several servers

Single-phase system- A system in which the **customer** receives service **from** only one station and then exits the system

Multiphase system A system in which the customer receives services from several stations before exiting the system.

Negative exponential probability distribution A continuous probability distribution often used to describe the service time in a queuing system

Arrival rate-The average number of customers arriving per time period

Service rate- The average number of customers that can be served per time period

Balking-The customer decides not to enter the waiting line.

Reneging-The customer enters the line but decides to exit before being served.

Jockeying-The customer enters one line and then switches to a different line in an effort to reduce the waiting time.

1.6. Scope of the study

This study is based on the data obtained from the Ethiopian civil aviation authority. It doesn’t include factors like type of aircraft, speed of the aircraft and the like. Moreover, it is related to delays because of queue.

2. **Literature review**

In the literature, there were developed some models in order to support and assist managers in making the best decisions on waiting lines (Pang, P., 2004), (Sweeney, D. et. al. 2010). In the management terminology, a waiting line is also called *the tail* and their characteristic concepts form *the queuing theory* (Shim, J., K., Siegel, J., G., 1999), (Williams, A., S., 2003). This theory is underlying the analysis of some communication, logistic, manufacturing and services systems (Bejan, A., 2007). The main advantage of queuing theory resides in determining very important information about waiting times, arrivals and service stations characteristics and about the systems discipline (Alecu, F., 2004).

Waiting line models consists of mathematical formulas and relations used to determine the operating characteristics of these lines. Among these features we mention (Williams, A., S., 2003):

The probability that there is no item in the system;

The average of the items in the waiting line;

The average of the existent items in the system

The items in the waiting line and the items being served);

The average time an item spends in the waiting line;

The average time an item spends in the system (consists of the waiting time besides the service time);the probability that an item has to wait for the service.

3. Research methodology

The study follows two phases: the project phase and the thesis phase.

As the research objective is describing the state of waiting line phenomena at Bole international Airport and maximizing the cost-effectiveness, the effectiveness of the selected solution is tested through simulations. The research focuses on the waiting phenomena and on Optimization techniques for operations scheduling using queue model. Here, a model built was finally compared with the real and conclusion can be derived accordingly.

To execute this some Methods used are:

1. Conducted a literature review
2. Observed the group (Ethiopian civil aviation) four hours per week for 3weeks, focusing mostly on conversations at team meetings, especially those conversations in which the group addresses changes to their work processes.
3. Interviewed team members to clarify and provide insight into traffic control and attempted to conduct these interviews shortly after conversations of interest. While the interviews were not be formal or structured, the kinds of questions asked include the following. The general strategy for the interviews is to start off with broad questions and follow up on the interviewee's responses, to capture her or his meanings and to avoid imposing my meanings on the interviewee.

3.1. Empirical Studies & Testing

The aiding framework for the design of queuing systems is developed in 4 stages including literature studies of background information, as illustrated in Conceptual Background, and then followed by a series of empirical studies and testing, which involve field study and interviews.

3.1.1. Empirical Study I: Field Study

Having to synthesize numerous existing literature studies, the obtained background knowledge of queuing has served as a conceptual basis to carry on to the next step of the development—to get a better understanding of how queue works and to obtain the general pattern of the use of various queue types in real life scenarios through real life observations.

3.1.2. Methods of field study

The field study was done through obtaining firsthand observations of all sorts of queuing scenarios that exist in real life. In each of these scenarios, observation was made specifically on the nature of different queue types, any design features of the waiting area, and the specific behavior of aircrafts which are waiting in a line and about to join the waiting line.

3•3. Waiting line Models

Mathematical equations that characterize waiting lines

Single-Channel, Single-Phase Models

The simplest, most basic of the waiting line structures illustrated in Figure 16.2 is the single-channel, single-phase model. There are several variations of the model that will be reviewed:

- All assume a Poisson arrival rate
- Variations to be reviewed use:
- exponential service times
- constant service times
- exponential service times with finite queue length
- exponential service times with finite calling population

The Basic Single-Server Model

The assumptions of the basic single-server model are

- Poisson arrival rate
- exponential service times
- first-come, first-served queue discipline
- infinite queue length
- infinite calling population

- λ = mean arrival rate
- μ = mean service rate

Formulas used for calculating the operating characteristics are shown in the following

3.3.1. Formulas for the Single-Server Model

Probability that no Aircraft is in the system: $P_0 = 1 - \frac{\lambda}{\mu}$

Average number of Aircrafts in waiting line(Lq) $L_q = \frac{\lambda^2}{\mu(\mu - \lambda)}$

Average number of Aircraft in the system(Ls) $L_s = L_q + \frac{\lambda}{\mu}$

Average time an aircraft spends in the waiting line $W_q = \frac{Lq}{\lambda}$

Average time Aircraft spends in the system(w) $W = W_q + \frac{1}{\mu}$

Probability of exactly *n* customers in the system: $P_n = (\frac{\lambda}{\mu})^n p_o$

Utilization rate $\rho = \frac{\lambda}{\mu}$

3.3.1.1Constant Service Times

The single-server model with Poisson arrivals and *constant service times* is used most frequently for systems with automated equipment and machinery. In the case of constant service times, there is no variability in service times.

- Constant service times occur with machinery and automated equipment
- Constant service times are a special case of the single-server model with general or undefined service times
- Operating characteristics for constant service times can be calculated with the formulas below:

3.3.1.2 Formulas for Constant Service Time Single-Server Model

Probability that no customers are in the system:

$$P_0 = 1 - \frac{\lambda}{\mu}$$

Average number of customers in the system:

$$L = L_q + \frac{\lambda}{\mu}$$

Average number of customers in waiting line:

$$L_q = \frac{\lambda^2}{2\mu(\mu - \lambda)}$$

Average time a customer spends in the system:

$$W = W_q + \frac{1}{\mu}$$

Average time a customer spends waiting in line to be served:

$$W_q = \frac{Lq}{\lambda}$$

3.3.1.3. Finite Queue Length

For some waiting line systems, the length of the queue may be limited by the physical area in which the queue forms; space may permit only a limited number of customers to enter the queue. Such a waiting line is referred to as a *finite queue*.

- A physical limit exists on the length of queue.
- M = maximum number in queue
- Service rate does not have to exceed arrival rate ($\mu > \lambda$) to obtain steady-state conditions.
- The operating characteristics of this variation of the single-server model can be calculated with the following formulas:

3.3.1.4 Formulas for Finite Queue Length Single-Server Model

Probability that no customers are in the system(p_o)

$$Po=\frac{1-\frac{\lambda}{\mu}}{1-(\frac{\lambda}{\mu})^{m+1}}$$

Probability of exactly *n* customers $P_n= (\frac{\lambda}{\mu})^n p_o$ for $n \leq m$

in the system: (p_n)

Average number of customers

in the system: (L) $$L= \frac{\frac{\lambda}{\mu}}{1-\frac{\lambda}{\mu}} - \frac{(m+1)(\frac{\lambda}{\mu})^{m+1}}{1-(\frac{\lambda}{\mu})^{m+1}}$$

Average number of customers in waiting line: $$L_q = L - \frac{\lambda(1-p_m)}{\mu}$$

Average time a customer spends in the system: $$W = \frac{L}{\lambda(1-pm)}$$

Average time a customer spends waiting in line to be served: $$W_q = W - \frac{1}{\mu}$$

Finite Calling Populations

In the single-server model with finite calling populations, the population of customers from which arrivals originate is limited, such as the number of police cars at a station answer calls.

- Arrivals originate from a finite (countable) population
- N = population size
- The operating characteristics for the single-server model with finite calling population can be computed with the formulas below.

3.3.1.5 Formulas for Finite Calling Population Single-Server Model

Probability that no customers are in the system: (p_o)

$$P_o = \frac{1}{\sum_{n=0}^{N} \frac{N!}{(N-n)!}\left(\frac{\lambda}{\mu}\right)^n}$$

Probability of exactly *n* customers in the system: (p_n)

$$Pn = \frac{N!}{(N-n)!}\left(\frac{\lambda}{\mu}\right) p_o$$

Average number of customers in the system:

$$L = L_q + (1 - P_0)$$

Average number of customers in waiting line:

$$Lq = N - \left(\frac{\lambda+\mu}{\lambda}\right)(1 - p_o)$$

Average time a customer spends in the system:

$$W_q = \frac{Lq}{\lambda(N-L)}$$

Average time a customer spends waiting in line to be served: $W_q = W - \frac{1}{\mu}$

3.3.2. Multiple-Channel, Single-Phase Models

An example of a multiple-channel, single-phase model is an airline ticket and check-in counter. A single line of customers queues to multiple servers in parallel. Its assumptions are:

- Two or more independent servers serve a single waiting line
- Poisson arrivals, exponential service, infinite calling population
- The number of channels x the service rate must exceed the arrival rate, $s\mu > \lambda$
- The operating characteristics for the multiple-server model can be computed using the following equations

Mathematical formulas for the Basic Multiple-Server Model

Probability that no customers are in the system:

$$P_o = \frac{1}{\left[\sum_{n=0}^{n=s-1} \frac{1}{n!}\left(\frac{\lambda}{\mu}\right)^n\right] + \frac{1}{s!}\left(\frac{\lambda}{\mu}\right)^s\left(\frac{s\mu}{s\mu - \lambda}\right)}$$

Probability of exactly *n* customers in the system:

$$p_n = \begin{cases} \frac{1}{s!s^{n-s}}\left(\frac{\lambda}{\mu}\right)^n P_0, & \text{for } n > s \\ \frac{1}{n!}\left(\frac{\lambda}{\mu}\right)^n P_0, & \text{for } n \leq s \end{cases}$$

Average number of customers in the system:

$$L = \frac{\lambda\mu(\lambda/\mu)^s}{(s-1)!(s\mu - \lambda)^2} P_0 + \frac{\lambda}{\mu}$$

Average number of customers in waiting line: $L_q = L - \frac{\lambda}{\mu}$

Average time a customer spends in the system: $W = \frac{L}{\lambda}$

Average time a customer spends waiting in line to be served: $W_q = W - \frac{1}{\mu}$

The performance of the service system is very poor if the waiting time is too long, and too many customers must wait.

4. Discussion and data Analysis

From the collected data of service time (see appendix) the following time (in minutes) are obtained.

5’	4’	6'	20'	4'	1'	27'
10’	8'	15'	5'	20'	23'	19'
21’	3'	3'	12'	18'	21'	11'
12’	12'	1'	1'	4'	7'	5'
10’	9'	11'	2'	16'	24'	13'
15’	5'	3'	9'	29'	25'	9'
4’	1'	19'	13'	1'	28'	18'
7’	9'	7'	2'	26'	8'	
17’	2'	5'	28'	3'	29'	

** Table 1.Group of service time (time to serve an aircraft)

In order to test the validity of the data using chi-square goodness –of –fit test the data is divided into bins of length one.

Number	Deviation(in minute)
1	[1,2]
2	[3,4]
3	5,6]
4	[7,8]
5	[9,10]
6	[11,12]
7	[13,14]
8	[14,15]
9	[16,17]
10	[18,19]
11	[20,21]
12	[22,23]
13	[24,25]
14	[26,27]
15	[28,30)

***. Table 2 The bins of the data

The next step is to find the relative frequency and cumulative probability distribution function (CDF) for the binned data.

Number	Deviation(in minutes)	Frequency	Relative frequency	CDF
1	[1,2]	8	0.131147541	0.131147541
2	[3,4]	8	0.131147541	0.262295082
3	5,6]	6	0.098360656	0.360655738
4	[7,8]	5	0.081967213	0.442622951
5	[9,10]	6	0.098360656	0.540983607
6	[11,12]	5	0.081967213	0.62295082
7	[13,14]	2	0.032786885	0.655737705
8	[15,16]	3	0.049180328	0.704918033
9	[17,18]	3	0.049180328	0.754098361
10	[19,20]	2	0.032786885	0.786885246
11	[21,22]	2	0.032786885	0.819672131
12	[23,24]	2	0.032786885	0.852459016
13	[25,26]	2	0.032786885	0.885245902
14	[27,28]	3	0.049180328	0.93442623
15	[29,30)	2	0.032786885	0.967213115

***Table 3.The relative frequency and CDF,of the service time

The plot of the data now can be sketched as:

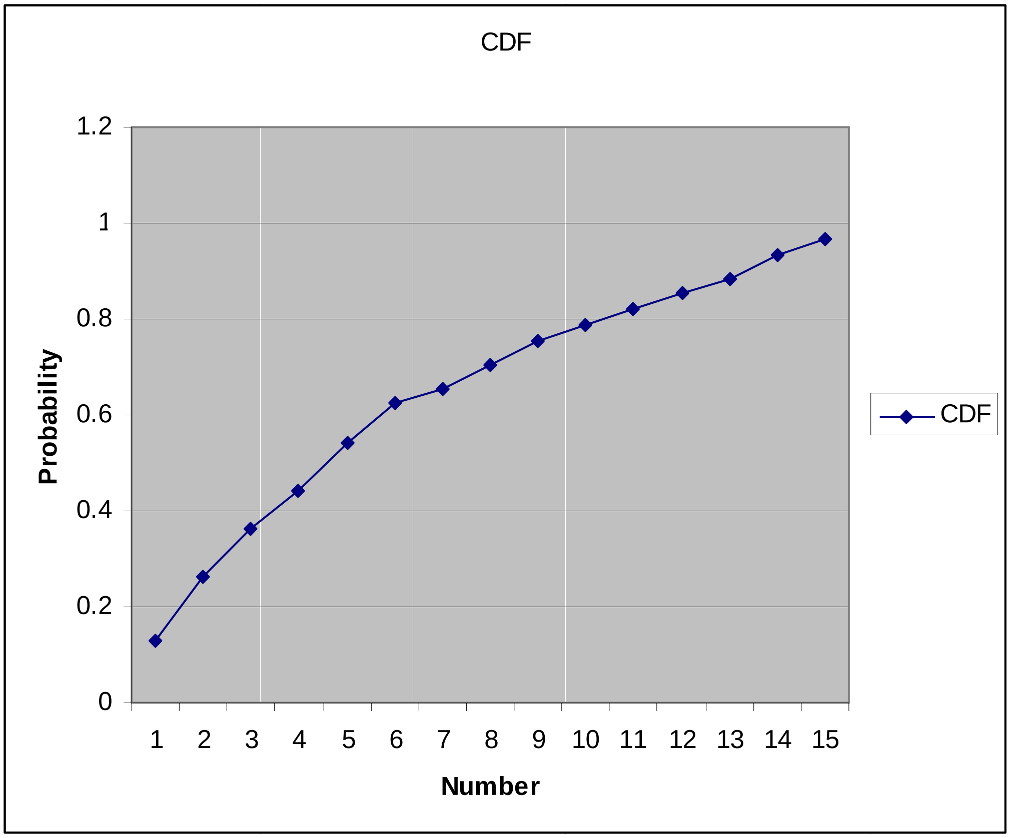

Figure 4. Graph of the Cumulative distribution function, CDF

Now the mean, t and the variance v of the empirical distribution can be estimated.

Let the number of bins be N in the data and if t_i is the mid points of the bin i then Mean, t = $\sum_1^N fiti$, and Variance V= $\sum_1^N fi(ti - t)2$ then we get: t= 7.57minutes (with tolerance)

The next step is checking the validity of this data using chi-square Goodness- fit-test. To do this the theoretical cumulative probability distribution has to be calculated first. A cumulative distribution function (CDF) give the probability than a random variable X is less that a given value x. (F(x) = Pr{X<=x}). An empirical distribution function is quite similar, the only difference being that we work from data rather than theoretical functions.

To compute the theoretical Cumulative distribution function (CDF, theoretical) Exponential distribution is used. From above mean, t= 7.57minutes (with tolerance).This implies Lamda=1/0.126= 8 service per unit time (with tolerance) for the hypothised exponential distribution and the associated theoretical cumulative distribution function is $f(t)=8(e^{-8t})$ .Now $F(t)= \int_0^T f(t)dt=1-(e^{-8T})$ where T is the mid value of the bins .calculating all the images of T under F(t) we have all the theoretical CDF. Now, the tabular representation is given below for both empirical and theoretical CDF.

Number	Deviation(in minutes)	Frequency	Relative frequency	Emerical,CDF	Theoretical,CDF
1	[1,2]	8	0.1311475	0.131147	0.1174
2	[3,4]	8	0.1311475	0.262295	0.253
3	5,6]	6	0.0983606	0.360655	0.3676
4	[7,8]	5	0.0819672	0.442622	0.4646
5	[9,10]	6	0.0983606	0.540983	0.5468
6	[11,12]	5	0.0819672	0.622950	0.616
7	[13,14]	2	0.0327868	0.655737	0.6752
8	[15,16]	3	0.0491803	0.704918	0.7251
9	[17,18]	3	0.0491803	0.754098	0.7673
10	[19,20]	2	0.0327868	0.786885	0.803
11	[21,22]	2	0.0327868	0.819672	0.8333
12	[23,24]	2	0.0327868	0.852459	0.8588
13	[25,26]	2	0.0327868	0.88524	0.8805
14	[27,28]	3	0.0491803	0.934426	0.8988
15	[29,30)	2	0.0327868	1.000000	0.9143

*** Table 4.Empirical and calculated (theoretical) CDF of the service time.

Their corresponding simulated plot of the theoretical and empirical distributions is:

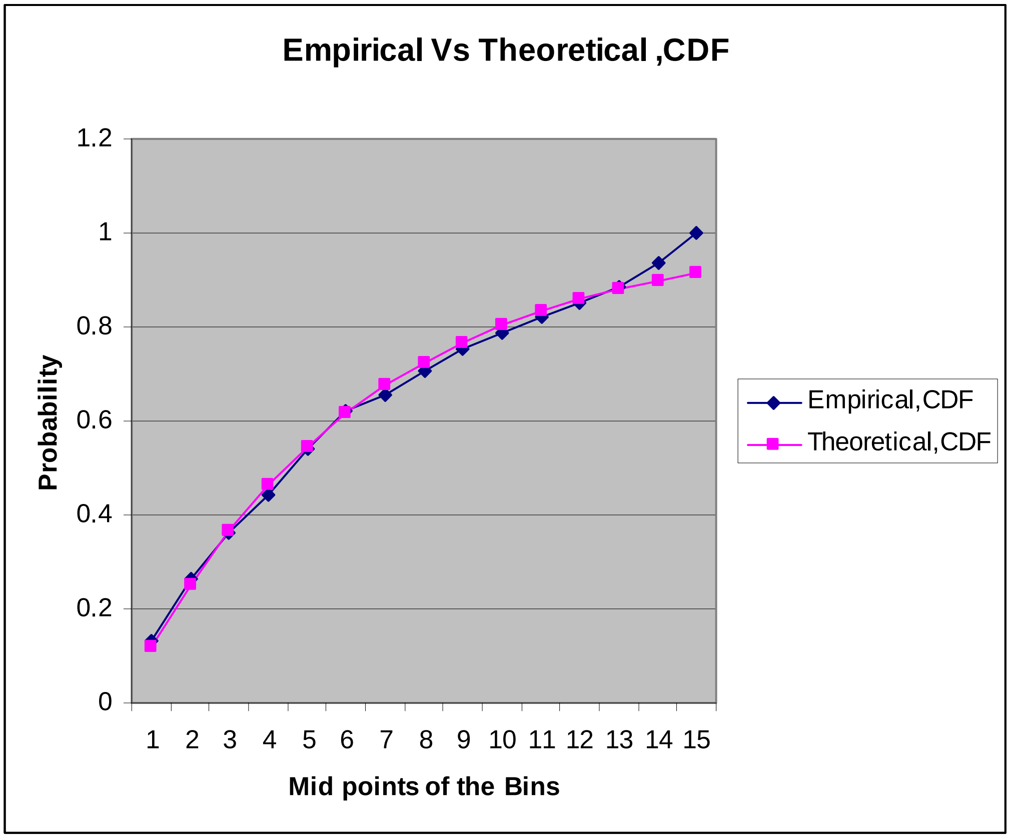

Figure 5. Simulated plot of Emperical ,CDF and Theoretical,CDF

From the plot it is automatically understood that the sample is drawn from the hypothesized exponential probability density function as the two CDF do not deviate excessively.

4.1. Results and Discussion

Performance Measurement

Before finding the performance measurements of Bole international airport let us first find the aircrafts waiting time.

To find the average waiting time of aircrafts the mean and standard deviation is required. What's nice about this method of finding the waiting line is that it does not assume a particular arrival rate or service distribution. All that is needed is the mean and standard deviation of the inter-arrival time and the service time.

. The service time is the amount of time that it takes to serve each customer.

Using the data collected let's, calculate the mean and standard deviation of the service time. Here, the following things are all identified.

1. Elements of a waiting line to be known to use queuing theory models

2. The basic structures of waiting lines.

3. Operating characteristics that are typically calculated when evaluating the performance of a service.

From statistics, mean = $\sum_{i=1}^{N} x_i / N$ where x_i is observed value (time) and N is the total number of observed aircrafts. As calculated above mean =7.57minutes.That means to serve an aircraft an average of 7.57 minutes is required.

Moreover, from the data (interview), Bole international airport entertains an average of 150 aircrafts per day (in 24 hrs). This means the arrival rate is 150/24= 6 aircrafts per hour.

The standard deviation is;

$$s = \sqrt{\frac{\sum_{i=1}^{N}(x_i - \overline{X})^2}{N-1}}$$

To find standard deviation let's bring the relative frequency of the data above;

N	xi	Mean	xi-mean
1	0.1311475	7.57	-7.438852459
2	0.1311475	7.57	-7.438852459
3	0.0983607	7.57	-7.471639344
4	0.0819672	7.57	-7.488032787
5	0.0983607	7.57	-7.471639344
6	0.0819672	7.57	-7.488032787
7	0.0327869	7.57	-7.537213115
8	0.0491803	7.57	-7.520819672
9	0.0491803	7.57	-7.520819672
10	0.0327869	7.57	-7.537213115
11	0.0327869	7.57	-7.537213115
12	0.0327869	7.57	-7.537213115
13	0.0327869	7.57	-7.537213115
14	0.0491803	7.57	-7.520819672
15	0.0327869	7.57	-7.537213115
		Sum	**-112.5827869**
		(Sum)*sum	12674.8839
		Sqrt(sum*sum)	112.582
		Standard deviation	=112.528/59=1.908169492

From the data the mean service rate, is equal to 7.57minutes=0.126 hrs.

λ = Aircrafts arrival rate = 150 aircrafts per day = 6 aircrafts per hour

μ = Aircrafts service rate = 1/0.126=8 aircrafts per hour

Utilization of the server is =Ω= λ/μ =6/8 =0.75.This means the runway is busy for 75% of the time.

The performance characteristics then become;

Probability that no Aircraft is in the system:	$P_0 = 1 - \frac{\lambda}{\mu}$	$P_0 = 1 - \frac{6}{8} = 0.25$
Probability of *4* Aircrafts in the system:	$P_n = \left(\frac{\lambda}{\mu}\right)^n P_0$ for $n \leq M$	$P_4 = (\frac{6}{8})^4 * P_0 = 0.0771$
Average number of Aircrafts in waiting line(Lq)	$L\,q = \frac{\lambda^2}{\mu(\mu-\lambda)}$	$Lq = \frac{6^2}{8(8-6)}$ = 2 Aircrafts
Average number of Aircraft in the system(Ls)	$L_s = Lq + \frac{\lambda}{\mu}$	$L_s = 2 + \frac{6}{8}$ = 3 Aircrafts
Average time an aircraft spends in the waiting line	$Wq = \frac{Lq}{\lambda}$	$W = \frac{2.25}{6}$ = 0.375 hrs.
Average time Aircraft spends in the system(w))	$W = Wq + \frac{1}{\mu}$	$W_q = 0.375 + \frac{1}{8} = 0.5$hrs
Utilization of the server	$\Omega = \frac{\lambda}{\mu} = \frac{6}{8} = 0.75$	

*** Performance evaluation

4.2. Discussion

Based on the results obtained above, aircrafts are likely to react differently to different scenarios, specifically to different queue types. Numbers of determinants to customers' decision making before and while queuing are being narrowed down into the following list through the analysis of all observations made:

☐ *Length of waiting line- As it is done above an average six aircrafts are in the system every hour*

☐ *Speed of delivery of service – Here also an aircraft elapses an average of 0.0771 hrs in the system.*

☐ *Queue layout:* Fairness vs. Speed- For the current existing case things are all manageable but the difficulty is when flight destinations increase.

☐ *Personal preferences:* choice, available time, - The runway is idle for 15% of the time and busy for 85% of the time.

☐ *Nature of services:* - Here the nature of service is very good for the existing situation .But difficult to serve more aircraft.

☐ *Number of attributes of service provided by a waiting line:* a line providing multiple attributes that provides only a single attribute of service(s)

Summary of solutions:

- P_0 = 0.1428, the probability of idle
- L_{q} =5 Aircrafts waiting

W_q = 0.8333hours= 50 minutes waiting for takeoff or landing.

- From the above computation, the facts that the average time of aircrafts waiting in line is 0.8333hrs.= 50 minutes & 85.714 % of the arriving aircrafts have to wait for service are indicators that something should be wanting line operation it is rational to focus on ways to improve the service rate. This can be achieved by either or both the case below.

A. Increase the mean service rate u.

Here, from our data the current service rate =7 aircrafts per hour & arrival rate =6 aircrafts per hours.

Assume our service rate is increased to 10 aircrafts /hr. then the operating characteristics become:

Probability that no Aircraft is in the system:	$P_0 = 1- \frac{\lambda}{\mu}$	$P_0 = 1-\frac{6}{10} = 0.400$
Average number of Aircrafts in waiting line(Lq)	$L\,q= \frac{\lambda^2}{\mu(\mu-\lambda)}$	$L_q = \frac{6^2}{10(10-6)}$ = 1Aircraft
Average number of Aircraft in the system(Ls)	$L_s = Lq + \frac{\lambda}{\mu}$	$L_s = 1 + \frac{6}{10}$ = 2Aircrafts
Average time an aircraft spends in the waiting line	$Wq = \frac{Lq}{\lambda}$	$W = \frac{1}{6}$ = 0.166hrs.
Average time Aircraft spends in the system(w))	$W = Wq+\frac{1}{\mu}$	$W_q = 0.166 + \frac{1}{10}$ = 0.266hrs
Utilization of the server	$\Omega= \frac{\lambda}{\mu} = \frac{6}{10} = 0.6$	* This means the runway is busy for 60% of the time.

**These manifests all the operating characteristics are improved by increasing the service rate. From the two operating characteristics, the following difference is observed.

Operating characteristics	Previous system	New system
Average no of customers in waiting line	2 air crafts	1 air craft
Average time an aircraft spend in the waiting line	22.5 minutes	10 minutes
Average time an aircrafts spend in the system	30 minutes	16 minutes
Probability that no air craft is in the system	0.25	0.400
Utilization rate	75%	60%

Table5: operating characteristic of single server with service rate, μ, six and ten.

B. Add one more new server (runway)

Add one more server with the same capacity so that more aircrafts can be served simultaneously. The performance characteristics for the multi-server then become.

Take $\lambda = A$ arrival rate= 6 aircrafts per hour, μ = service rate= 8 aircrafts per our and

s= no of server=2. Applying this

Probability that no customers are in the system:

$$P_o = \frac{1}{\left[\sum_{n=0}^{n=s-1} \frac{1}{n!}\left(\frac{\lambda}{\mu}\right)^n\right] + \frac{1}{s!}\left(\frac{\lambda}{\mu}\right)^s \left(\frac{s\mu}{s\mu - \lambda}\right)}$$

$p_o = 0.45$

Average number of customers in the system:

$$L = \frac{\lambda\mu(\lambda/\mu)^s}{(s-1)!(s\mu-\lambda)^2} P_0 + \frac{\lambda}{\mu}$$

L= 1 aircraft

Average number of customers in waiting line:

$$L_q = L - \frac{\lambda}{\mu}$$

Lq=1- 0.75=0 aircrfts

Average time a customer spends in the system:

$$W = \frac{L}{\lambda}$$

W=1/6=0.1666hrs=10 minutes

Average time a customer spends waiting in line to be served:

$$W_q = W - \frac{1}{\mu}$$

Wq=(0.16667-0.125)hrs= 3 minutes

Utilization rate

$$P= \frac{\lambda}{s\mu} = \frac{6}{(2)(8)} =0.375$$

The performance of the service system is very poor if the waiting time is too long, and too many customers must wait.

The summarized difference between the single server and multiple servers is given below.

Operating characteristics	Single server	Multiserver
Average no of customers in waiting line	2 air crafts	1 air craft
Average time an aircraft spend in the waiting line	22.5 minutes	3 minutes
Average time an aircrafts spend in the system	30 minutes	10 minutes
Probability that no air craft is in the system	0.25	0.45
Utilization rate	75%	37.5%

***Table 6.Operating characteristics of single server v_s that of Multiserver

*Finally, comparing all the three techniques the following difference is observed on performance characteristics.

Operating characteristics	Single server With service rate,6	Single server with service rate,10	Multiserver with service rate,6
Average no of customers in waiting line	2 air crafts	1 air craft	1 air craft
Average time an aircraft spend in the waiting line	22.5 minutes	10 minutes	3 minutes
Average time an aircrafts spend in the system	30 minutes	16 minutes	10 minutes
Probability that no air craft is in the system	0.25	0.400	0.45
Utilization rate	75%	60%	37.5%

***Table 7. Comparison of the solution of the system.

5. Conclusions and recommendations

In the design of a queuing system, two primary parameters are involved – the number of servers and the number of lines, which are influenced not only by cost and capacity, but also by aircraft perceptions. It is apparent that the greater the number of servers and lines the greater the cost and volume required to accommodate such queue setting, whereas customers' perceptions influence mainly the decision to the number of lines. When aircrafts arrive at a system that has a single long queue, they may decide not to join and leave the system. This situation may be enhanced by a more effective distribution of customers into different queues, or a better design of the queue layout such as curving the waiting line back and forth into zigzag forms so as to reduce space that are being occupied. Both methods serve to cut distance into sections, and by doing so, customers' perception of distance will be altered. Yet more generally, unless there is a need to segment aircrafts into multiple lines, a single line system is always preferable because it preserves a high level of social justice and relatively lower ability in social comparisons. While a good layout design does not only reduce traffic congestion and waiting time, smooth operations, but also more importantly, it improves changes customers' satisfaction.

5.1. Recommendations

The queuing model presented in this thesis provides a solution to the waiting line management problem of minimizing the amount of delay incurred by an airline. The practicality of the model is also important to note because it can be solved very quickly using commercial integer programming solvers.

Therefore, it is recommended that Ethiopian civil aviation has to investigate the cost to be minimized (profit obtained) by revising flight schedules, and changing its queue discipline first come first served which does not favor profit maximization. ,For example if Fokker 50,which consumes less fuel compared to Boeing 763 requested first to land ,it is advisable to allow the Boeing to land first as it takes more fuel in waiting in air though it requests latter. Finally implement the schedules and he disciplines to maximize its profit.

5.2. Conclusions

Queuing models have found widespread use in the analysis of service facilities, production and many other situations where congestion or competition for scarce resources may occur. concepts of queuing models, and linear programming, and in some cases a mathematical analysis, can be used to estimate the performance measures of a system. The key operating characteristics for a system are:

(a) Utilization rate,(b) percent idle time, (c) average time spent waiting in the system and in the queue, (d) average number of aircrafts in the system and in the queue , and (5) probabilities of various numbers of aircrafts in the system.

In overall, the waiting line models play a key role in highlighting the operations effectiveness and hence the need of improving their characteristics. It is rational to analyze and decide on the changes regarding the waiting line configuration (flight schedule). In order to improve the operations within the waiting line, improving the service rate is therefore crucial. This is possible by adopting one or both solutions listed below:

The increase of the average service rate μ – this is possible by either redesigning the waiting line or using current advanced technologies.

The addition of new Servers(Runways) – so that more aircrafts can be served simultaneously.

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7. Appendix

Start, service Time	Aircraft type (name)	Aircraft SPEED	Destination	End, service time	Service time
21:40	B763	B535DT	VTBS	21:45	5’
21:45	B763	B535DT	VIDP	21:55	10’
20:10	B343	R611DT	LFPG	21:31	21’
20:35	A343	R611DT	EDDF	20:47	12’
20:55	B752	R611DT	LIRF	22:05	10’
6:30	B763	A408	FNLU	6:45	15’
4:50	FK50	DCT/BD	HABD	4:54	4’
10:15	FK50	DCT/AM	HAAM	10:22	7’
10:15	Q400	CT/5M	HAJM	10:32	17’
7:15	B737	G650D	HKJK	7:19	4’
9:30	Q400	B535DL	HDAM	9:38	8'
3:55	Q400	DG/GN	HAGN	3:58	3'
20:15	B737	G650D	HKJK	20:27	12'
20:15	B738	B535DP	LLBG	20:24	9'
20:25	B737	A408D	HRYR	20:30	5'
20:35	A343	R611DT	EDDF	20:36	1'
21:00	B737	R611DT	H55J	20:19	9'
21:10	B752	R611DT	LFPG	21:12	2'
6:45	B763	4B736	GABS	6:51	6'
6:30	B752	A408D	FCBB	6:45	15'

Start ,service Time	Aircraft type	Aircraft SPEED	Destination	End , service time	Service time
6:00	C550	DCT/LL	HALL	6:01	1'
3:55	Q400	DCT/GN	HAGN	4:06	11'
6:05	B757	GR50D	FAJJ	6:08	3'
11:30	Q400	B535DF	H55J	11:49	19'
7:45	B737	B535DP	OMDB	7:52	7'
8:54	Q400	B535DL	HDAM	8:59	5'
6:15	B752	B736D	FGSL	6:35	20'
7:15	Q400	G650D	HKMO	7:20	5'
7:00	B737	B736D	DNAA	7:12	12'
7:15	B763	R611DT	EDDF	7:37	1'
7:35	B737	B535DL	HDAM	7:37	2'
7:15	B737	G650D	HKJK	7:24	9'
6:35	B763	A408D	FCBB	6:48	13'
22:45	MD11	R611DT	HECA	22:47	2'
21:45	B752	B535DT	VIDP	22:13	28'
22:15	A330	B535DP	OYSN	22:16	1'
21:55	B763	R611DT	EGLL	21:59	4'
21:55	B757	R611DT	LIRF	22:15	20'
21:25	B763	B535DT	VTBS	21:43	18'
21:00	B737	R611DT	HSSS	21:04	4'
20:40	A343	R611DT	EDDF	20:56	16'
21:20	B752	B535DP	OEJN	20:49	29'

Start ,service Time	**Aircraft type**	**Aircraft SPEED**	**Destination**	**End, service time**	**Service time**
6:30	B763	G650D	FVHA	7:06	26'
6:30	B737	A408D	HBBA	6:33	3'
6:20	B763	A408D	FCBB	6:43	23'
6:15	B752	B736D	DXXX	6:36	21'
7:30	B737	G650D	FWKI	7:37	7'
7:30	B738	G650D	HTDA	7:54	24'
20:00	B737	G650D	HKJK	20:25	25'
20:30	Q400	G650D	HRYR	20:58	28'
19:45	B738	R611DT	HECA	19:53	8'
19:45	Q400	G650D	HUEN	20:14	29'
19:15	B763	B535DP	OMDB	19:37	27'
20:45	B737	R611DT	HSSS	21:04	19'
20:15	B737	B535DP	OERK	20:26	11'
19:15	B763	B535DP	OMDB	19:20	5'
19:45	B763	R611DT	HECA	19:58	13'
8:15	Q400	B535DF	HSSJ	8:24	9'
6:15	B757	B736D	DXXX	6:33	18'